1 The Method in a Nutshell

Moment
projection onto basis

Normalization
tfo. into standard position

Moment Invariant
independent from certain tfo.

2 Moments

Moments are the projections onto a function space basis. We use the monomial basis.

Definition 1. For $a \in \mathbb{N}$ and a three-dimensional vector field $v : \mathbb{R}^3 \to \mathbb{R}^3$ with compact support, the $a$-th order vector moment tensor $M_{(a)}$ is defined as

$$M_{(a)} = \int_{\mathbb{R}^3} x^a \cdot v(x) \, dx.$$ 

Figure 1: The first basis vector fields of the monomial basis.

Theorem 1. The vector moment tensor of order $n$ is a contravariant tensor of rank $n-1$ and weight 1.

3 Normalization

We put the pattern into a pre-defined standard position.

The transformations for flow field pattern recognition with respect to which we normalize, take the shape

$$v'(x) = v(R(x)) + t,$$

with velocity $v \in \mathbb{R}^3$, background flow $f \in \mathbb{R}^3$, and rotation $R \in SO_3$.

Figure 2: Scalar normalization.

Figure 3: Flow field normalization

4 Moment Invariants

Instead of comparing flow fields, we compare their normalized moments. The moments that were not used in the normalization process form a complete and independent set of invariants.

The similarity of a position $s$ and a scale $s$ in a flow field to a pattern independent of background flow, velocity and orientation is given by the reciprocal of the Euclidean distance in the moment feature space

$$S(s, x) = \sum \frac{1}{|m_{f(i)}^p - m_{i}^p|^2},$$

Figure 7: More complicated structures, like the quadrupole ($D$), need higher order moments to be detected.

Algorithm 1 Flow field pattern recognition

Input: flow field, pattern, maximal moment order
1. calculate the moments of the pattern
2. normalize the moments of the pattern
3. for all positions $s$ and all scales $s$ do
   a. calculate the moments of the field at point $s$ with size $s$
   b. normalize them
   c. 4D similarity field: $S(s, x) = \sum \frac{1}{|m_{f(i)}^p - m_{i}^p|^2}$
   d. 3D density field: draw sphere around $s$ with radius $s$ and value $S(s, x)$
4. end for

Output: density field.

Moment invariants are powerful and flexible flow field descriptors.

5 Results

We constructed a dataset with different flow patterns with varying positions, sizes, velocities, background flows, and orientations to give a nice overview on the behavior of the moment invariants. It is described in detail in the following table and illustrated in Figure 4.

<table>
<thead>
<tr>
<th>ID</th>
<th>Pattern</th>
<th>Basic pattern</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0,0)</td>
<td>a very weak source</td>
<td></td>
</tr>
<tr>
<td>(1,0,2)</td>
<td>an oval vortex with core line along the z-axis, drawn out along the x-axis</td>
<td></td>
</tr>
<tr>
<td>(2,2,-2)</td>
<td>a dipole in the x-y plane</td>
<td></td>
</tr>
<tr>
<td>(2,2,-2)</td>
<td>a vortex added to a quadrupole in the x-y plane</td>
<td></td>
</tr>
<tr>
<td>(1,2,0)</td>
<td>a short vortex with core line along the y-plane</td>
<td></td>
</tr>
<tr>
<td>(2,2,2)</td>
<td>a long vortex with small diameter and its core line along the y-axis</td>
<td></td>
</tr>
</tbody>
</table>

Figure 4: The transfer function of the volume rendering.

Figure 5: Illustration of the flow structures in the data set. The color bar represents the velocity.

Figure 6: The query pattern: a vortex.

Figure 8: The output of the algorithm: The similarity of the moments is encoded in the density and the size of the match is encoded in the radius of the spheres. The resulting scalar field is visualized using volume rendering. Additionally, the probability of a point to be a seedpoint of a streamline is the similarity of the moments.