Bild- und Signalverarbeitung
UNIVERSITÄT LEIPZIG

Moment Invariants for 3D Flow Fields

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1 The Method in a Nutshell

Moment

projection onto basis

Normalization

trfo. into standard position

Moment Invariant

independent from certain trfo.

2 Moments

Moments are the projections onto a function space basis. We use the monomial basis.

Definition 1. *For $n \in \mathbb{N}$ and a three-dimensional vector field $v : \mathbb{R}^3 \to \mathbb{R}^3$ with compact support, the n-th order* **vector moment tensor** *$M_{i_0 i_1 \ldots i_n}$ is defined as*

$$M_{i_0 i_1 \ldots i_n} = \int_{\mathbb{R}^3} x_{i_1} \ldots x_{i_n} v_{i_0}(x)\, \mathrm{d}^3 x.$$

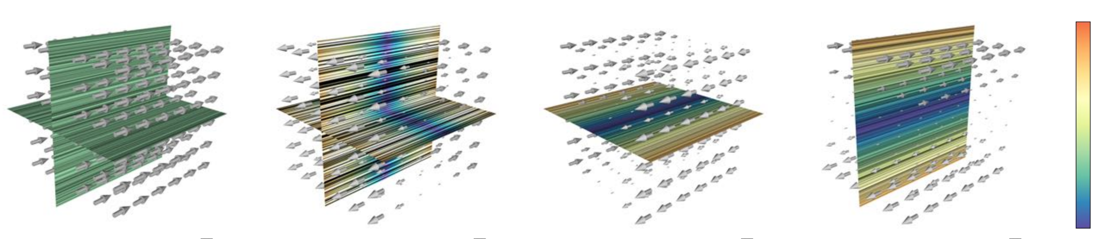


Figure 1: The first basis vector fields of the monomial basis.

Theorem 1. *The vector moment tensor of order n is a contravariant tensor of rank $n+1$ and weight 1.*

3 Normalization

We put the pattern into a pre-defined standard position.

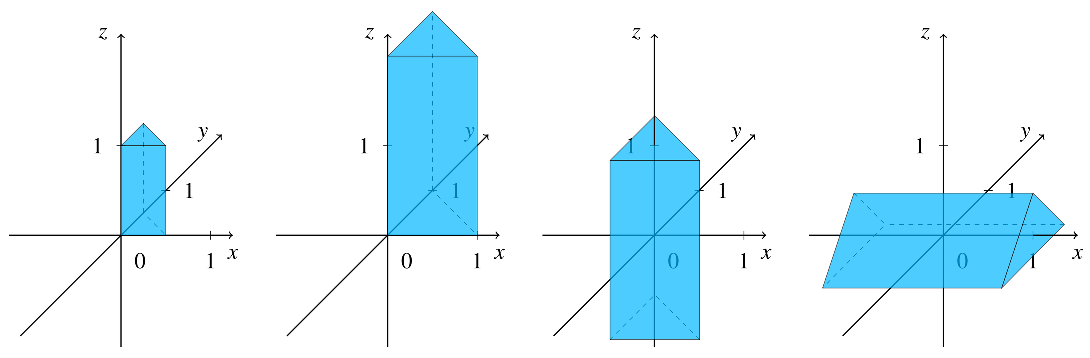


Figure 2: Scalar normalization.

The transformations for flow field pattern recognition with respect to which we normalize, take the shape

$$v'(x) = sRv(R^{-1}x) + t,$$

with velocity $s \in \mathbb{R}$, background flow $t \in \mathbb{R}^3$, and rotation $R \in SO_3$.

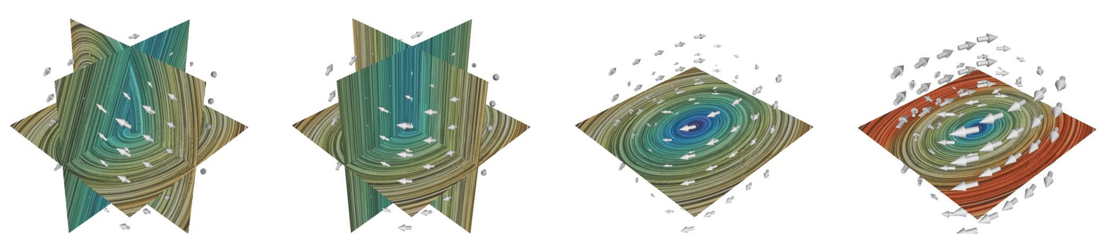
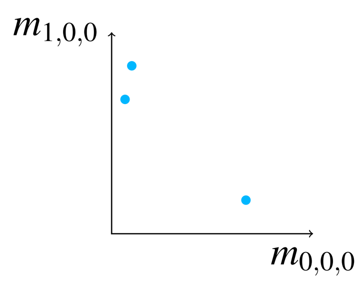


Figure 3: Flow field normalization.

4 Moment Invariants

Instead of comparing flow fields, we compare their normalized moments. The moments that were not used in the normalization process form a complete and independent set of invariants.



The similarity of a position x and a scale s in a flow field to a pattern independent of background flow, velocity and orientation is given by the reciprocal of the Euclidean distance in the moment feature space

$$S(x,s) = \Big(\sum_{p,q,r} |m^{pattern}_{p,q,r} - m^{field}_{p,q,r}|^2\Big)^{-1}.$$

Algorithm 1 Flow field pattern recognition

Input: flow field, pattern, maximal moment order

1: calculate the moments of the pattern
2: normalize the moments of the pattern
3: **for** all positions x **and** all scales s **do**
4: calculate the moments of the field at point x with size s
5: normalize them
6: 4D similarity field: $S(x,s) = (\sum_{p,q,r} |m^{pattern}_{p,q,r} - m^{field}_{p,q,r}(x,s)|^2)^{-1}$
7: 3D density field: draw sphere around x with radius s and value $S(x,s)$
8: **end for**

Output: density field.

Moment invariants are powerful and flexible flow field descriptors.

5 Results

We constructed a dataset with different flow patterns with varying positions, sizes, velocities, background flows, and orientations to give a nice overview on the behavior of the moment invariants. It is described in detail in the following table and illustrated in Figure 4.

ID	Position	Basic pattern
	(0,0,0)	a very weak source
(A)	(2,-2,2)	a sink
(B)	(1,0,2)	an oval vortex with core line along the z-axis, drawn out along the x-axis
(C)	(2,2,-2)	a bipole in the x-y-plane
(D)	(2,-2,-2)	a vortex added to a quadrupole in the x-y-plane
(E)	(-2,-2,-2)	a saddle
(F)	(-2,2,-2)	a short vortex with core line along the axis $(0,-1,1)^T$
(G)	(-2,0,2)	a long vortex with small diameter and its core line along the y-axis

Figure 4: The transfer function of the volume rendering.

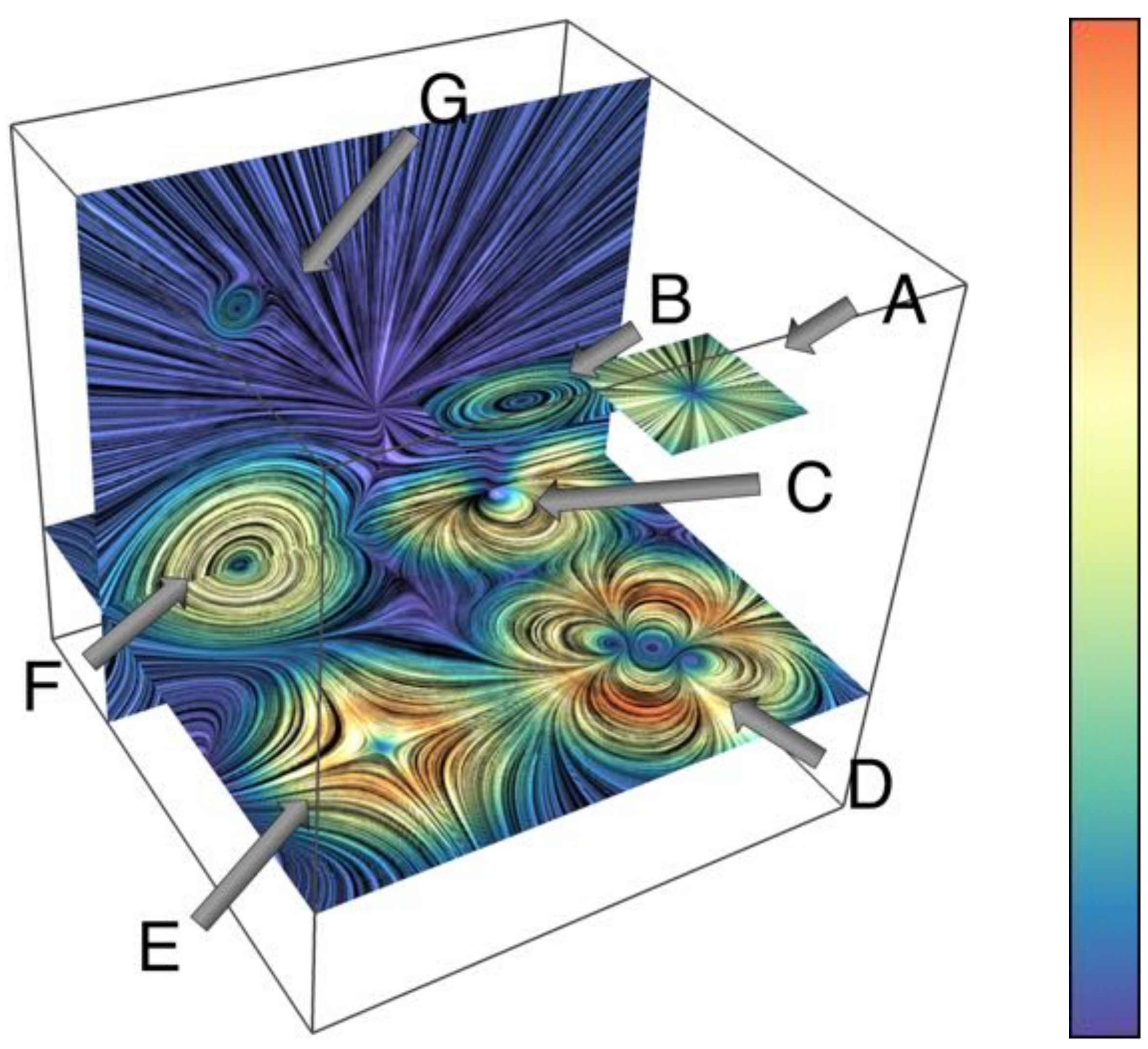


Figure 5: Illustration of the flow structures in the data set. The color bar represents the velocity.

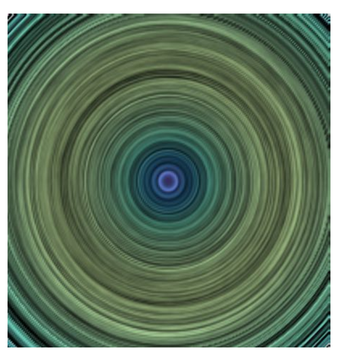


Figure 6: The query pattern: a vortex.

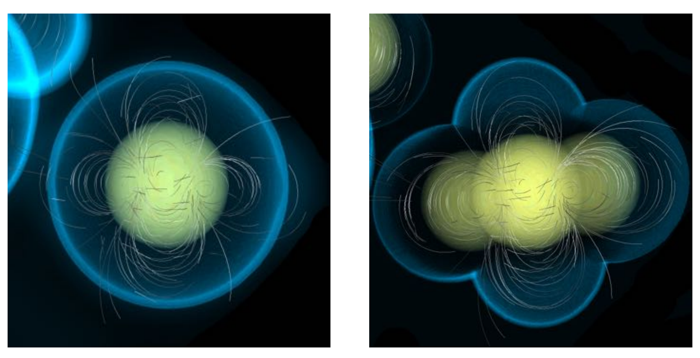


Figure 7: More complicated structures, like the quadrupole (D), need higher order moments to be detected.

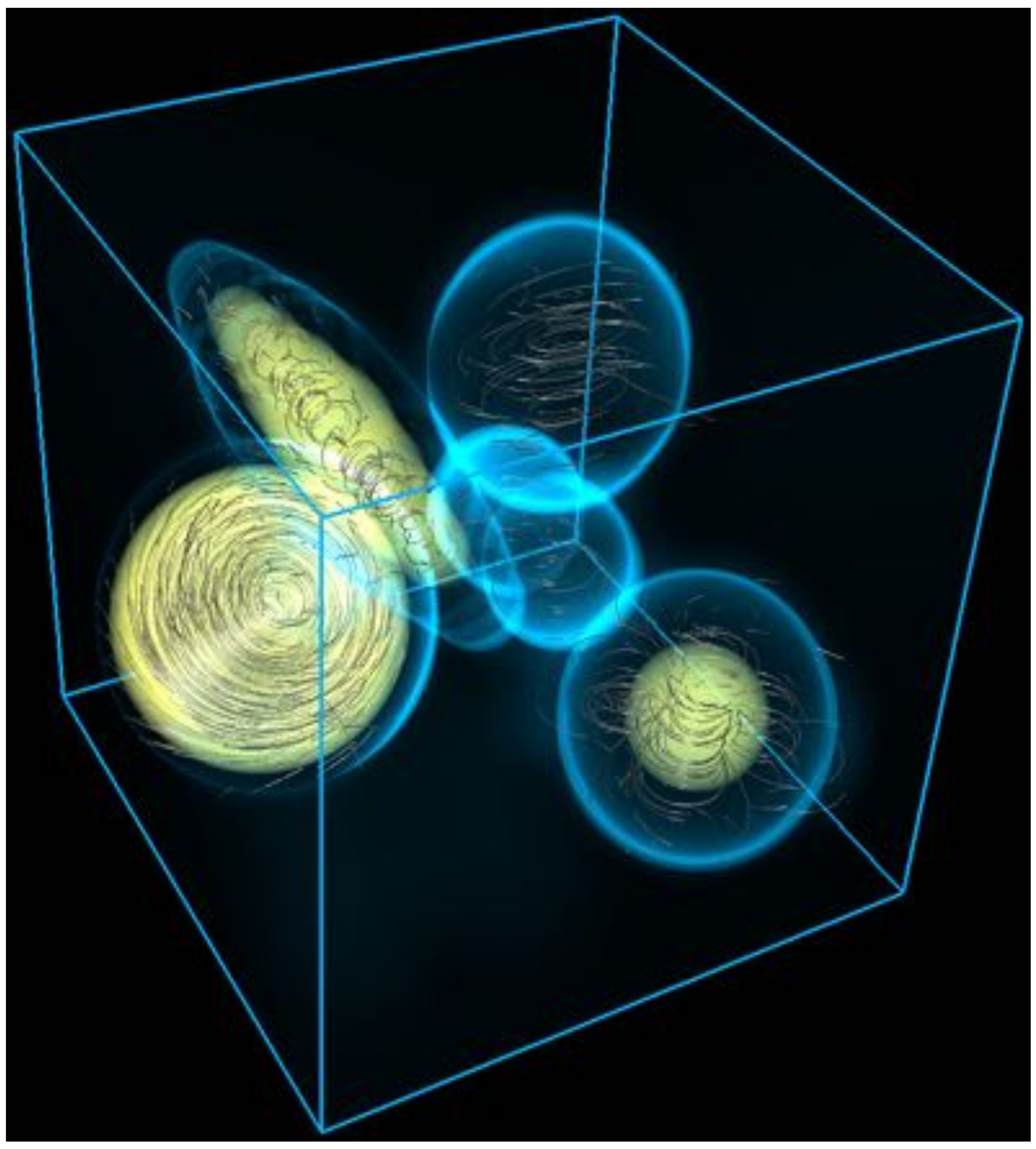


Figure 8: The output of the algorithm: The similarity of the moments is encoded in the density and the size of the match is encoded in the radius of the spheres. The resulting scalar field is visualized using volume rendering. Additionally, the probability of a point to be a seedpoint of a streamline is the similarity of the moments.