RECOGNITION OF PATTERNS IN VECTOR FIELDS
BY GAUSSIAN-HERMITE INVARIANTS

Bo Yang
School of Automation,
Northwestern Polytechnical University
Xi’an Shaanxi, P.R. China

Jitka Kostková, Jan Flusser, Tomáš Suk
Institute of Information Theory and Automation
Czech Academy of Sciences

Roxana Bujack
Data Science at Scale Team,
Los Alamos National Laboratory

1. Numerical Precision

2. Template matching in a gradient field

3. Template matching in a fluid flow field

1. Vector field
\[ f(x) = (f_1(x), f_2(x)) \]

2. Total rotation of vector field
\[ f'(x) = R_\alpha f(R_{-\alpha} \cdot x) \]

3. Vector field in complex plane
\[ f(x, y) = f_1(x, y) + if_2(x, y) \]

4. Gaussian-Hermite polynomials
\[ H_n(x) = 2xH_{n-1}(x) - 2(n - 1)H_{n-2}(x) \]

5. Gaussian-Hermite moments
\[ \hat{n}_{pq} = \frac{1}{\sigma \sqrt{\pi (p + q)! 2^{p+q}}} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H_p(x, \sigma) H_q(y, \sigma) f(x, y) dx dy \]

6. Complex Gaussian-Hermite moments
\[ d_{pq} = \sum_{k=0}^{p} \sum_{j=0}^{q} \binom{p}{k} \binom{q}{j} (-1)^{q-j} j^{p+q-k-j} \hat{n}_{k+j,p+q-k-j} \]

7. Invariants to total rotation
\[ \Phi(p, q) \equiv d_{pq} g_{q,p}^{p-q+1} \]

Thanks to the Czech Science Foundation (Grant No. GA15-16928S) and the Grant Agency of the Czech Technical University (Grant No. SGS15/214/OHK4/3T/14) for funding.
Thanks to the National Natural Science Foundation of China (Grant No. 61502389) and the Fundamental Research Funds for the Central Universities (Grant No. 31020152Y047) for funding.
Thanks to Professor Mario Hlawitschka for providing the von Kármán vortex street data.