GOL: A Framework for Building and Representing Ontologies

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Abstract. The present paper is a contribution to the emerging field of ontological engineering. We discuss the basic principles of ontology from an axiomatic-deductive point of view. We clarify the notion of an ontology and the connection between ontology and knowledge systems. An ontology comprises ontologically basic relations such as membership, part-whole and inheritance. The ontological basic entities are divided into Urelemente and classes, the Urelemente are in turn either individuals or universals. Foremost among the individuals are substances, moments and sitoids. We outline an ontology representation language called GOL, for (General Ontological Language), and we compare GOL to other similar languages developed for purposes of ontology.

1 Introduction

In recent years AI researchers have been developing ideas and methods in ontology that play a significant role in work on knowledge representation and reasoning. Ontologies are generally held to provide standardized definitions of the terms used to represent knowledge. An ontology can thus be considered as an integral part of a declarative knowledge representation language. During the last few years a growing interest in the design, use, and sharing of ontologies has been observed. Work in this area obviously incorporates work on formal issues of knowledge representation with the development of practical implemented systems. Nowadays, this new emerging field of ontological engineering is a craft rather than a science. One reason is that there is no foundation and definition of the main concepts and methodologies that should govern the development of ontologies. In the present paper the ontological foundations of the modelling, acquisition and representation of knowledge will be discussed.

The paper is organized as follows. Section 2 describes several phases of knowledge acquisition and section 3 we will discuss the notion of an ontology and its relation to knowledge systems. Section 4 is devoted to the analysis of important ontologically basic relations and entities, in particular to the set-merological aspects of the world. Section 5 describes the first draft of an ontology representation language extending KIF and similar languages such as F-logic. Axiomatic fragments of mereotopology are discussed in section 7.
2 Logical Principles of Knowledge Acquisition

Common-sense knowledge and reasoning is at the center of AI because human cognitive agents always start out from a situation in which the information available has a common-sense character. Although mathematical models of the traditional kind are contained, at least partially, in common sense, it seems to be impossible to reduce common sense to the usual mathematical theories which utilize only set-theoretical tools. This is for reasons of principle: set theory captures only a part of the ontology of the world. In spite of this ontological restriction of mathematics, its formal methods represent an ideal model for any science, in particular for the evolving science of axiomatic formal ontology.

2.1 The Axiomatic-Deductive Method

In the sequel the terms *formal theory* and *formal knowledge base* are used as synonyms. A formal theory is a set of formalized propositions. The axiomatic method contains several principles used for the development of formal knowledge bases and reasoning systems aiming at the foundation, systematization and formalization of a field of knowledge associated with a part or dimension of reality.

The axiomatic method deals with the specification of concepts and is motivated, as we conceive it, by the following considerations. A formal knowledge base includes, on the one hand, primitive notions, defined notions, and definitions, and on the other hand axioms, theorems, and proofs. It would be ideal if one were able to explain explicitly the meaning of every notion occurring in whatever is the relevant domain and to justify each propositions in succession. When one tries to explain the meaning of a term, however, one necessarily uses other expressions, and, in turn one has to explain these expressions. If, now, one wishes to avoid entering into a vicious circle, then one has to resort yet further terms, and so on. We have thus the beginning of a process which can never be brought to an end. The situation is quite analogous for the justification of the statements asserted within a knowledge base: for in order to establish the validity of a statement, it is necessary to refer back to other statements, which leads again to an infinite regress.

The axiomatic-deductive method contains the principles necessary to solve this problem. When we set out to assemble in a systematic way the knowledge we have in regard to a given field of knowledge, then we can distinguish, first of all, a certain small group of concepts in this field that seem to be understandable or intelligible in and of themselves. The expressions in this group we call primitive or basic, and we employ them without formally explaining their meanings via explicit definitions. Examples are the concepts of identity or of part. At the same time we adopt the principle of not employing any other term taken from the field under consideration unless their meaning have first been determined with the help of the basis notions and of such expressions of the field whose meaning have been previously explained. The sentence which determines the meaning of a term in this way is called an explicit definition.
How, then, can the basic notions be described, how can their meaning be characterized? Given the basic terms, we may construct more complex sentences which may be understood as descriptions of certain formal interrelations between them. Some of these statements are chosen as axioms; we accept them as true without in any way establishing their validity by means of a proof. By accepting such sentences as axioms we assert that the described interrelations are considered to be valid and at the same time we define the given notions in a certain sense implicitly; i.e. the meaning of the basic terms is to some extent captured and constrained by the axioms. On the other hand, we agree to accept any other statement as true only if we have succeeded in establishing its validity from the chosen axioms via admissible deductions. Statements established in this way are called proved statements or theorems.

The method of establishing a body of knowledge relating to a given field in accordance with these principles is called the **axiomatic-deductive method**. An axiomatic-deductive system is a set of propositions in which each proposition is either one of the set of initial propositions (an axiom or “highest-level hypothesis”) or it is a proposition (a “lower-level hypothesis”) deduced from the set of initial propositions according to logico-mathematical principles of deduction. Some of the propositions of the system may be propositions about observations which are directly testable against experience.

### 2.2 Levels of a Knowledge Base

A knowledge base usually contains different (sometimes hidden) levels of generality. Thus, a knowledge base will make use of a basic logic which provides the principles of deduction. The basic logic includes all the deductive principles of the system. None of the latter is specific to the system itself and the deductive power of the system is achieved only with the addition of the system’s axioms.

The axioms can be classified into three main groups, one group consisting of those axioms required for basic logic. These logical axioms are true in every possible world. General ontological axioms are concerned with axioms about the ontologically basic relations, which will be studied in section 4. They describe those laws of the ontologically basic entities and relations which are true in every part of the world in which those entities exist and those relations obtain. General ontological axioms present what is sometimes called the *top-level ontology*. Finally, domain-specific axioms are tailored to a given concrete area of the world.

In summary, we distinguish the logical level, the general ontological level, and the domain specific level. We give some examples of axioms belonging to these different levels.

**Logical Axioms**

\[ \forall x (P(x) \lor \neg P(x)) \]
\[ \exists x (D(x) \rightarrow \forall y D(y)) \]
\[ \exists x \forall y D(x, y) \rightarrow \forall y \exists x D(x, y). \]
General ontological axioms

Axioms of set theory
\[ \forall xy \exists z (\forall u (u \in z \iff u = x \lor u = y)) \].

Part-whole relation
\[ \forall xyz (x < y \land y < z \rightarrow x < z) \].

Identity
\[ \forall xy (x = y \rightarrow y = x) \]

Axioms of mereotopology:
"Every line is a boundary of a surface"

Domain-specific Axioms

- Axioms of genetics,
- Axioms of physics etc.

3 Ontologies and Knowledge Bases

In this section we will define more precisely the terms 'ontology' and 'conceptualization', used in an informal manner in the previous section. The term 'ontology' became popular within the knowledge engineering community with the work of Gruber[21], but its usage has remained unclear. There are several ways to make this notion more precise. We have taken as reference for our analysis the papers [21], [23], [36] and have added some further clarifications.

3.1 The Notion of Ontology

There are several approaches to defining the notion of an ontology. Aristotle defined metaphysics (a term which we take as synonymous with the term 'ontology') as the science of being as such, as contrasted with the special sciences, each of which investigates a certain class of things and their properties. Ontology considers "all the species of being qua being and the attributes which belong to being qua being" (Metaphysics, IV,1). This level of description we call General Ontology, in contrast with the various Domain Specific, Special or Regional Ontologies. In the sequel we assume the following definition of ontology as a research area:

Ontology is the science which is aimed at the systematic and axiomatic development of the theory of all forms and modes of being, [12].

Such a science is obviously a foundation for the development of those knowledge bases called "ontologies" in the AI-community. There are however several interpretations of this term in use among AI researchers and we have selected here some typical examples.

1. An ontology is an explicit specification of a conceptualization.
2. An ontology is a representation of a conceptual system via a logical theory.
3. An (AI)-ontology is a theory of which entities can exist in the mind of a knowledgeable agent.

4. Ontologies are agreements about shared conceptualizations.

The interpretation 1. was proposed in [19] as a definition of what an ontology is for the AI-community. The interpretation 2 is formulated in [23] and is essentially the same as 1. The interpretation 3 refers to ontology as the theory of being from the subjectivistic perspective. The agreements mentioned in interpretation 4 can be understood as specifications of conceptualizations supported and accepted by the majority of the people working in a certain domain.

Here, in contrast, we assume the point of view of realism, i.e. the position that the kind of things we are speaking about have objective existence, so that there are for our purpose only what we might call good conceptualizations. We propose, therefore, a revision of Gruber’s interpretation [19], reinterpreting it also from the point of view of the axiomatic method.

3.2 Conceptualizations

In current practice the term ‘ontology’ is used ambiguously either to refer to a symbolic system or to its semantical counterpart. In [17] Genesereth and Nilsson define a conceptualization as a structure \((D, R_1, \ldots, R_n)\) representing a set \(D\) of objects and a number of relations \(R_1, \ldots, R_n\). They use set-theoretical terms to specify such systems. Obviously, such a conceptualization is a (somehow) vague description of a part of the world. In which language (apart from natural language) may such a system be specified? We will show that to answer this question requires a conceptualization language which contains not only set theory but also some other fundamental categories. Guarino [23] criticizes Genesereth-Nilsson’s notion of a conceptualization because it refers exclusively to “ordinary mathematical relations on \(D\)”, i.e. to extensional relations. Guarino, in contrast, wants to focus on the meaning of these relations. He accordingly refers to intensional or conceptual relations, and tries to capture the latter by using total functions from possible worlds into sets. A conceptualization then is defined as a tuple \((D, W, \mathcal{R})\), where \(\mathcal{R}\) is a set of conceptual relations which is such that every \(R \in \mathcal{R}\) is considered as a total function assigning to every world \(w \in W\) a relation \(R(w)\). But, this total function and the relations \(R(w)\) are themselves extensional objects and one could question whether we gain by this construction some insight in the term “intensional”. The point is that not every entity in the world is a set and thus, as we shall see, set theory is not able to capture the world’s full ontology.

Another point concerns how Guarino describes and specifies this conceptualization \(\mathcal{C} = (D, W, \mathcal{R})\). In [23] it is implicitly suggested that \(\mathcal{C}\) should be specified in set theoretical terms. As an example of such a specification we may consider the field of real numbers which may be defined (up to isomorphism) in set theory. In [23], therefore, set theory is considered as a kind of representation language for specifying conceptualizations. The notion of a conceptualization as introduced by Guarino in [23] can be equivalently described by a class of first-order structures of a certain signature satisfying the additional condition that
their domains coincide. Thus, the approach in [23] can be summarized as follows. Let there be given a class \( \mathcal{K} \) of structures representing our intuitions about a given domain. We say that a set-theoretical formula \( \varphi \) partially describes \( \mathcal{K} \) if \( \mathcal{K} \subseteq \text{Mod}(\varphi) \), where \( \text{Mod}(\varphi) \) is the class of structures specified by \( \varphi \). Therefore, conceptualizations are partially specified by set-theoretical formulas.

In contradistinction to this approach we want to use a more general ontological language, which includes set theory as a proper part. This representation language should express partial descriptions of reality directly. Such a language should have the expressive power to describe those entities whose ontology cannot be fully captured by set theory. Furthermore, we have to use certain ontologically basic relations and predicates such as inherence, part-whole, causality, time, space, universals, and situations. Some of these relations will be investigated in section 4.

### 3.3 Formal Knowledge Bases

Usually, a formal knowledge base is built on a conceptualization which is specified in set-theoretical terms. We adopt a realistic view of the ontology of truth, based on a suitable correspondence theory. On the one hand, we assume that propositions are bearers of the properties of truth and falsehood; on the other hand, we assume the existence of entities in reality in virtue of which true propositions are true. These entities we call truth-makers. We use here the notion of truth-maker in the broader sense of being a part of the world in which a proposition can be satisfied, i.e. can be made true by certain entities contained in it. The simplest form of a truth-relation was formalized by Tarski, [37], but, his correspondence theory is not fully developed because it is restricted to the investigation of set-theoretical surrogates -models and conceptualization- and excludes the ontology of the world itself.

An important problem in specifying a knowledge representation language is to develop an ontological foundation which may serve as the language's formal semantics. The current knowledge representation languages are mainly based on classical set theory. This means that the terms used in these languages are reduced to set-theoretical notions. An ontologically adequate language has to go beyond set theory by including terms that refer directly to those objects in the world which are not sets, for example entitites as trees, houses, water, people, processes, qualities, etc.

### 4 Ontologically Basic Entities

In this section we will introduce and discuss the ontologically basic entities and certain basic relations between them. These relations and entities correspond to the most general categories of the world, such as space, time, reality, substance, property, mind, matter, states and the like. It is a permanent concern of philosophers whether these categories are absolute, complete and universal. Aristotle lists ten categories: substance, quantity, quality, relation, place, time, posture,
state, action, and passion; but he was not absolutely convinced that the classifi-
cation was definite. Our analysis is similar to that in [30], but adds the idea of
large ontological regions such as the region described by set and class theory.

Our main distinction is between Urelemente, classes, and sets. We assume
the existence of urelements, sets, and classes in the world and presuppose that
both the impure sets and the pure sets constructed over the urelements belong
to the world.

Sets, classes and Urelements. Urelements are entities which are not sets or
classes. The entities of the world are classified with respect to their type. Sets
and urelements are entities of type 0, and $C_0$ is the class of all entities of type 0.
Let $\tau_1, \ldots, \tau_n$ be types, and $C_{\tau_i}$ the class of all classes of type $\tau_i$, respectively.
Then $C_{\tau_1, \ldots, \tau_n}$ is the class of all classes whose $n$ arguments are classes of types
$\tau_1, \ldots, \tau_n$, respectively. A class is of finite type if it can be generated by a finite
number of such iteration steps. Let $C_{FT}$ be the class of all classes of finite type.
In our class hierarchy $C_{FT}$ is the top-most node.

Neither the membership relation nor the subclass relation can unfold the
internal structure of urelements. There thus arise the following questions:

1. Which categories of urelements are there?
2. Which formal relations hold among the urelements and between the urelem-
ents and the non-urelements?

We classify urelements into individuals and universals, and we classify indi-
viduals further into substances, moments and situoids. At the bottom we have
the class $U$ of urelements thought of as a realm of concretely existing things in
the world.

Individuals and Universals. An individual is a single thing thought of in con-
trast to universals. A universal is an entity that can be instantiated by a number
of different individuals. The individuals covered by a universal are thus similar
in some respect. There are different approaches to universals: Platonism is the
position that universals exists independently and before individuals (ante rem);
Aristotelian is the position that universals exist in the individuals (in re) and
thus not independently from them; conceptualism is the view that universals are
the reflections of the propensity of the mind to group things together (post rem)
or that universals are somehow abstracted from individuals. In our approach we
assume that the universals exist in the individuals (in re) but not independently
of them; our attitude is thus Aristotelian in spirit.

The world is divided into several ontological regions: one of them, according
to the view here defended, is the area of classes and sets. For every universal
$U$ there is a set $\text{Ext}(U)$ containing all instances of $U$ as elements. Concerning
the regional ontology of sets our position is similar to that of K. Gödel,[25], who
believed that sets represent an aspect of objective reality.

We take for granted axioms to the effect that the class of urelements is the
disjoint union of the class of individuals and the class of universals, and that
there is no entity being both an individual and a universal.
The individuals are further subcategorized into moments, substances, chronoids, topoids and situations.\footnote{This classification is, of course, tentative.}

**Substances.** Substance is that which can exist by itself, or does not need a subject in order to exist, in the way that properties need objects; hence a substance is that which bears properties. Examples are an individual person, a house, the moon, a car.

**Moments.** Here, an accident is a property of a thing which is not a part of the essence of the thing; something it could lose or have added without ceasing to be the same thing or the same substance. We use the term “moment” in a more general sense and do not distinguish between essential and inessential moments. Moments include individual qualities, actions and passions, a flash, a handshake, thoughts and so on; moments thus comprehend what are sometimes referred to as ‘events’. If a substance loses a moment in this more general sense then this, may change its essence. Moments have in common that they all depend on individual substances. Moments can be classified with respect to several aspects. The arity of a moment is the number of its arguments; unary moments are called qualities or attributes. Relational moments are non-unary moments having a fixed finite number of arguments, and adic moments have a non-determinate number of arguments without fixed upper bound. The basic ontological relation of inheritance, denoted by $i$, connects moments with substances.

**Sitoids.** A sitoid is, intuitively, a part of the world that can be comprehended as a coherent whole and does not need other entities in order to exist. An example is: John’s kissing of Mary in a certain environment. This sitoid contains the substances ‘John’ and ‘Mary’ and a relational moment ‘kiss’ which connects them. These entities in isolation do not yet form a sitoid, we have to add an environment consisting of further entities and a location in order to get a comprehensible and coherent whole: John and Mary may be sitting on a bench or may be walking through a park. The notion of being comprehensible as a coherent whole will be elucidated formally in terms of a grasping relation between sitoids and certain universals.

**Configurations.** A configuration $c$ in the sitoid $s$ is defined as some result of taking a collection of substances and other individuals occurring in $s$ and adding moments and material relations from $s$ which serve to glue them together. A configuration $c$ over the subinterval $i \leq chr(s)$ is a configuration in the sitoid $s \downarrow i$.

**Situations.** Situations are special types of sitoids: they are sitoids at a time, so that they represent a snap-shot view of some part of the world. Situations can be conceived as projections of sitoids onto times or equivalently as sitoids with an atomic framing chronoid.

**Chronoids and Topoids.** Chronoids can be understood as “durations”, and topoids as “space regions” with a certain mereotopological structure.
proach to space and time is inspired by the ideas of F. Brentano [8] who developed and elaborated Aristotle’s sketchy remarks in the *Physics* about boundaries and continua. Chisholm in [10],[11] has made a first step towards interpreting Brentano’s idea in a formal manner, and B. Smith in [32] has continued and extended this work by presenting a strict axiomatic system about mereotopology.

5 Ontology of Relations

Relations are entities which glue together the things of the real world. Every relation has a number of *relata* or *arguments* which are connected or related by it. The number of a relation’s arguments is called its arity. We admit the possibility of anadic relations, i.e., relations with an indefinite number of arguments. Relations can be classified also according to the types of their relata. There are relations between sets, between individuals, and between universals, but there are also *cross-categorical* relations for example between urelements and sets or between sets and universals.

We divide relations into two classes, called *material* and *formal*, respectively. The relata of a material relation are mediated by individuals which are called *relators*. Relators are individuals which connect entities; kisses, contracts, conversations, for example, are individuals generating relators which connect individual persons. One has to distinguish between the relator itself and its foundation. A conversation is the foundation for the relator of being connected by a *conversation*. A formal relation is a relation which holds between two or more entities directly—without any further intervening individual. Examples are: larger than, part-of, different from, dependent on.

5.1 Holding Relation and Facts

One important formal relation is called the *holding relation*. If \( r \) is a relator connecting the entities \( a_1, \ldots, a_n, n \geq 1 \), then we say that \( r, a_1, \ldots, a_n \) (in this order) stand to each other in the holding relation, symbolized by \( h(r, a_1, \ldots, a_n) \). The fact that \( h \) holds directly suffices to block the obvious regress which would arise if a new relator were needed to tie \( h \) to \( r, a_1, \ldots, a_n \), and so on.

If \( r \) connects (holds of) the entities \( a_1, \ldots, a_n \), then this yields a new individual which is denoted by \( \langle r : a_1, \ldots, a_n \rangle \). Individuals of this latter sort are called *material facts*. Note that the \( a_i \) are not necessarily individuals, for example the fact *Mary is speaking about humanity* can be represented as the fact *<speaking, Mary, humanity>* where the material relation “speaking” (in the sense of an individual speech act) connects *Mary* with the universal *humanity*. Material facts are in every case constituents of sitouids, and sitouids are collection of facts into wholes. A material fact \( \langle r : a_1, \ldots, a_n \rangle \) has a duration, which depends on the lifetime of the relator \( r \). We write \( \langle r : a_1, \ldots, a_n; i \rangle \) if \( i \) is a chronoid which is a part of the lifetime of \( r \), i.e., this fact exists at least during the interval \( i \). Individuals of the form \( \langle r : a_1, \ldots, a_n; i \rangle \) are called temporialized material facts.
5.2 Relator Universals and Relation Universals

A relator universal is a universal whose instances are relators. For every relator universal \( R \) there exists a class of facts, denoted by \( \text{facts}(R) \), which is defined by the instances of \( R \) and their corresponding arguments. We assume the axiom that for every relator universal \( R \) there exists a factual universal \( \mathcal{F}(R) \) whose extension equals the class \( \text{facts}(R) \). Take, for example, the relator universal \( U \) whose instances are individual kisses. Then we may form a factual universal \( \mathcal{F}(U) \) having the meaning \( A \text{ person } a \text{ kisses } a \text{ person } b \) whose instances are all facts of the form \( \langle k : a, b \rangle \), where \( k \) is an individual kiss and \( a, b \) are individual persons (\( k, a \) and \( b \), here, are variable terms). The factual universal \( \mathcal{F}(R) \) is the basis for two material relations \( R_1(F), R_2(F) \) whose instances are sequences of entities. \( R_1(F), R_2(F) \) are considered as universals of special type which are called relation universals. In our example they are defined as follows: \([a,b] : R_1(F) \iff \exists k \langle k : u_k \wedge \langle k, a, b \rangle : F(R) \rangle \). The instances of \( R_2(F) \) are sequences with an additional component being a chronoid: \([a,b,i] : R_2(F) \iff \exists k \langle k : u_k \wedge \langle k, a, b \rangle : F(R) \rangle \wedge i \leq \text{lifetime}(k) \rangle \). There are sub-universals \( F(U, J, M) \) of \( F(U) \), say, corresponding to the meaning: \( \text{John kisses Mary} \), whose instances are all facts of the form \( \langle k : J, M \rangle \) where \( J, M \) are the individuals \( \text{John} \) and \( \text{Mary} \). Natural-language sentences of the form \( A \text{ man } a \text{ kisses } a \text{ woman } b \text{ or } \text{John } a \text{ kisses } \text{Mary} \) can be interpreted as referring to factual universals.

5.3 Formal Relations

A formal relation is a relation which holds between two or more entities directly - without any further intervening individual. We consider them similar as material relations - as a kind of universal called relation universal - whose instances are sequences. Note that the components of these sequences are not necessarily individuals. If \( R \) is a formal relation and \([a,b] : R \) then \( \langle R : a, b \rangle \) is called a formal fact. Extensional relations are sets (or set-theoretical classes) of sequences. Obviously, every extensional relation is formal. We assume the axiom that for every formal relation \( R \) there is a set-theoretical class \( \text{ext}(R) \) being the extension of \( R \). A extensional function is defined as a set-theoretical class of finite sequences of entities, one for each combination of possible components. In each sequence, the initial elements are the arguments, and the final element is called the value.

5.4 Basic Relations

We can distinguish the following basic ontological relations, which are needed to glue together the entities mentioned above. The first and most familiar is that of membership, denoted by \( \in \). Then come the part-of relations, denoted by \( < \) and \( \leq \) (for proper and reflexive part-of). We assume that the part-of relations \( <, \leq \) have individuals in both arguments.

Inference. The phrase “inference in a subject” can be understood as the translation of the Latin expression \textit{in subjecto esse}, in contradistinction to \textit{de subjecto}
 dici, which may be translated as “predicated of a subject”. The inheritance relation \( x - y, u \) - sometimes called ontic predication - glues moments to the substances which are their bearers. For example it glues your smile to your face, or the charge in this conductor to the conductor itself.

**Relativized Part-Whole.** The ternary part-whole relation \( <(x, y, u) \) has the meaning: “\( u \) is a universal and \( x \) is a part of \( y \) relative to \( u \)”. Briefly, if \( x \) is a \( u \)-part of \( y \) in this sense, then \( x \) and \( y \) are parts of instances of the universal \( u \) and \( x \leq y \). We propose the following axiom: for every universal \( u \) there are universals \( u_1, \ldots, u_n \) such that \( <(x, y, u) \) implies that \( x, y \) are instances of one of the \( u_i \)'s and every instance of some of the \( u_i \)'s is part of an instance of \( u \).

Consider the following example, taken from the domain of biology. Let \( u_T \) be the biological universal whose instances are those organisms called trees. Then \( <(x, y, u_T) \) describes the part-whole relation which imposes upon the parts it recognizes a certain granularity, the granularity of whole trees. A biologist is interested in describing the structure of trees only in relation to parts of a certain minimal size. Thus she is not interested in atoms or molecules. There is a finite number of universals \( \{u_1, \ldots, u_n\} \) by which the biologically relevant parts of trees are demarcated. All such parts of trees are either instances of some \( u_i \), \( 1 \leq i \leq k \), or they can be decomposed into a finite number of parts, each of which satisfies this condition. Examples of relevant \( u_i \) would be branch of a tree, leaf of a tree, trunk of a tree, root of a tree, and so on.

**Instantiation.** The symbol \( :: \) denotes the instantiation relation. Its first argument is an individual, and its second a universal. If \( x :: u \), then \( u \) is a certain time- and space-independent pattern of features and \( x \) is an individual in which this pattern of features is realized. The symbol \( : \) denotes instantiation of a relation universal. Its first argument is a list of entities, and its second a relation universal. Note, that the components of the list are not necessarily individuals.

**Containment.** The containment relation \( \triangleright \) holds between the constituents of a situoid and the situoid itself. The constituents of a situoid \( s \) include, among other entities, the pertinent substances and the moments inhering in them. But also facts and configurations are constituents of situoids. Note, that not every part of a constituent of a situoid \( s \) is contained in \( s \).

**Framing.** Every situoid, for example the fall of a stone in a certain environment, consumes an amount of time and occupies a certain space. The binary relation of framing \( \sqsubset \) glues chronoids or topoids to situoids. We presume that every situation is framed by a chronoid and a topoid. The relation \( x \sqsubset y \) is to be read: ‘the chronoid (topoid) \( x \) frames the situation \( y \)’. Obviously, \( \sqsubset \) is a formal relation (no further entity is needed to link the chronoid with the situoid it frames). Let \( s \) be a situation, then \( chr(s) \) denotes the chronoid framing \( s \); \( tp(s) \) denotes the topoid framing \( s \).

**Location.** The binary relation \( occ(x, y) \) describes a fundamental relation between substances and topoids. \( occ(x, y) \) can be read: the substance \( x \) occupies the topoid \( y \) (roughly: \( x \) is located in \( y \)).
Grasping. The relation $gr(s,u)$ has the meaning: $s$ is a situoid and $u$ is a universal associated with $s$. These universals determine which material relations and individuals occur as constituents within a given situoid and thus which granularities and viewpoints it presupposes.

6 Outline of a Representation Language

In this section we will outline an ontology representation language using the concepts which were expounded in the preceding sections.

6.1 Syntax and Axioms

The elementary version of our intended representation language GOL (General Ontological Language) is formalized in a first-order language.

Alphabet.

$$<\text{variable}> ::= x_1 | x_2 | \ldots | x_n | \ldots (\text{variables of type 0})$$

Logical Connectives

$$<\text{logfunctor}> ::= \land | \lor | \neg | \rightarrow | \leftrightarrow | \forall | \exists$$

Basic Vocabulary

$$<\text{baspred}> ::= \text{Ur} | \text{Set} | \text{Ind} | \text{Mom} | \text{Chron} | \text{Top} | \text{Subst} | \text{Univ} | \text{RUniv}$$

$$<\text{basuniv}> ::= \text{Substance} | \text{Time} | \text{Space}$$

$$<\text{basrel}> ::= \epsilon | :: | :: | :: \text{ } | < | \subseteq | = | <\text{occ}| gr | >$$

The basic vocabulary is denoted by Bas.

User-defined Vocabulary.

$$<\text{indconst}> ::= \text{a letter denoting an individual}$$

$$<\text{extrelconst}> ::= \text{a letter denoting an extensional relation}$$

$$<\text{univconst}> ::= \text{a letter denoting a universal}$$

$$<\text{relunivconst}> ::= \text{a letter denoting a relation universal}$$

A user-defined vocabulary is called an ontological signature and is denoted by $\Sigma = (I, \text{ExtR}, U, RU)$.

Expressions.

Expressions are terms and formulas of the first order predicate calculus.

$$<\text{term}> ::= <\text{variable}> | <\text{constant}>$$

$$<\text{atomform}> ::=$$

$$<\text{term}> = <\text{term}> | <\text{term}> \neq <\text{term}> | \text{Un}(<\text{variable}>)) |$$

$$\text{Set}(<\text{term}>)) | \text{Ind}(<\text{term}>)) | \text{Univ}(<\text{term}>))) |$$

$$\text{Mom}(<\text{term}>)) | \text{Sub}(<\text{term}>)) | \text{Chron}(<\text{term}>)|$$

$$\text{Top}(<\text{term}>)) | \text{Subst}(<\text{term}>)) |$$

$$<\text{term}> \in <\text{term}> | <\text{term}> :: <\text{term}>)) |$$
<form> ::= 
  <atomform> | <form> ∧ <form> | <form> ∨ <form> | <form> → <form> | <form> ←→ <form> | ¬ <form> | 
  ∀ <variable> <form> | ∃ <variable> <form>

The language includes an axiomatization capturing the semantics of the ontologically basic relations. We don’t present the axiomatization in full detail, but illustrate the main groups by selecting some typical axioms. We introduce three groups of axioms whose union are the axioms $Ax(GOL)$ for the language $GOL$.

Let be $Ax(GOL) = \text{Logical axioms} \cup \text{Axioms about Bas} \cup \text{Axioms about } \Sigma$.

**Logical Axioms**

(a) Axioms of predicate logic

1. $\forall x (A \to B) \to (\forall x A \to B)$
2. $\forall x (A \to B) \to (A \to \forall x B), \ x \text{ not free in } A$
3. $\forall x A \to A(x/t), \ t \text{ is a term being free for } x$
4. $\exists x A(x) \leftrightarrow \neg \forall x \neg A(x)$

(b) Axioms of identity

1. $\forall x(x = x)$
2. $\forall xy(x = y \to y = x)$
3. $\forall x y z(x = y \wedge y = z \to x = z)$
4. $\forall x_1 x_2 \ldots x_n y(x_k = y \to
   F(x_1, x_2, \ldots, x_{k-1}, x_k, \ldots, x_n) \leftrightarrow F(x_1, x_2, \ldots, x_{k-1}, y, \ldots, x_n))$, for every formula $F$.

**Axioms about Basic Ontology**

(a) Sort and Existence Axioms

1. $\exists x (Set(x)),$
2. $\exists x (Ur(x))$
3. $\forall x (Set(x) \lor Ur(x))$
4. $\neg \exists x (Set(x) \land Ur(x))$
5. $\forall x (Ur(x) \leftrightarrow \text{Ind}(x) \lor \text{Univ}(x))$
6. $\neg \exists x (\text{Ind}(x) \land \text{Univ}(x))$

(b) Axioms about sets including ZF

1. $\forall u \exists x (Set(x) \land x = \{u, v\})$
2. $\{\phi^{Set} \mid \phi \in ZF\}$, where $\phi^{Set}$ is the relativization of the formula $\phi$ to the basic symbol $Set(x)$.
(c) Axioms about Moments and Substances

1. $\forall x (\text{Subst}(x) \rightarrow \exists y (\text{Mom}(y) \land i(y, x)))$
2. $\forall x (\text{Mom}(x) \rightarrow \exists y (\text{Subst}(y) \land i(x, y)))$
3. $\forall xyz (\text{Mom}(x) \land i(x, y) \land i(x, z) \rightarrow y = z)$

(d) Axioms about Part-Whole

Definitions,

$\text{ov}(x, y) = \exists z (z \leq x \land z \leq y)$, (overlap)

$x \leq y = \forall x \forall y (x \leq y \land y < x)$ (reflexive part-whole).

1. $\forall xy (x < y \rightarrow \text{Ind}(y) \land (\text{Ind}(x) \lor \text{Set}(z)))$
2. $\forall xy (\text{Set}(x) \land \forall z (z \in x \rightarrow z < y) \rightarrow x \leq y)$
3. $\forall x (\neg x < x)$
4. $\forall xyz (x < y \land y < z \rightarrow x < z)$
5. $\forall xy (\forall z (z < x \rightarrow \text{ov}(z, y)) \rightarrow x \leq y)$
6. $\forall xyz (x = y \rightarrow \text{Univ}(y) \land x < y)$
7. $\forall x y z u (\neg (x, y, u) \land (y, z, u) \neg (x, z, u))$

(e) Axioms about chronoids and topoids.

1. $\forall x (\text{Sit}(x) \rightarrow \exists (\text{Chron}(t) \land t \sqsubseteq x))$
2. $\forall x (\text{Sit}(x) \rightarrow \exists (\text{Top}(t) \land t \sqsubseteq x))$
3. $\forall x (\text{Chron}(x) \rightarrow \exists (\text{Top}(t) \land t \sqsubseteq x))$
4. $\forall x (\text{Top}(t) \rightarrow \exists (\text{Top}(t) \land t \sqsubseteq x))$

(f) Axioms about situations

Definitions.

$\text{Cont}(s) = \{ m \mid m \vdash s \}$

$s \sqsubseteq t = \forall s (\text{Cont}(s) \subseteq \text{Cont}(t))$

1. $\forall x (\text{Sit}(x) \rightarrow \exists (\text{Subst}(s) \land \text{Mom}(t) \land s \vdash t \land x < t))$
2. $\forall x (\text{Mom}(x) \rightarrow \exists (\text{Sit}(s) \land x \vdash s))$
3. $\forall x (\text{Subst}(x) \rightarrow \exists (\text{Sit}(s) \land x \vdash s))$
4. $\forall x (\text{Sit}(x) \land \text{Sit}(y) \rightarrow \exists (\text{Sit}(z) \land x \sqsubseteq z \land y \sqsubseteq z))$
5. $\neg \exists x (\text{Sit}(x) \land \forall y (\text{Sit}(y) \rightarrow x \sqsubseteq y))$

Axioms about $\Sigma$

1. $\text{Univ}(U)$ for every $U \in U$
2. $\text{Set}(R)$ for every $R \in RU$
3. $\text{Set}(R)$ for every $R \in \text{ExtR}$
4. $\text{Ind}(c)$ for every $c \in K$

A knowledge base about a specific domain with respect to the signature $\Sigma$ is determined by a set of formulas from GOL($\Sigma$) which are not basic axioms.
6.2 Semantics of GOL

Model-Theoretic Semantics. Let \( \Sigma \) be an ontological signature. An abstract \( \Sigma \)-interpretation is a first-order structure \( \mathcal{W} = (W, \text{Bas}^\delta, \Sigma^\delta) \), where \( W \) is a set and \( \Sigma^\delta \cup \text{Bas}^\delta \) are interpretations of the symbols from \( \Sigma \cup \text{Bas} \) in the set \( W \). Furthermore, \( \mathcal{W} \) satisfies the axioms \( \text{Ax}(\text{GOL}) \). On the other hand, there is the real world \( \mathcal{R} \) which is not a set; \( \mathcal{W} \) can be understood as a set-theoretical reflection of \( \mathcal{R} \), and we assume that every axiom from \( \text{Ax}(\text{GOL}) \) is true in \( \mathcal{R} \).

We use the notation \( \mathcal{R} \models \phi \) having the meaning: the sentence \( \phi \) is true in \( \mathcal{R} \); this is abbreviated by \( \models \phi \).

Global Semantics. The global semantics interprets the constants in \( \text{Bas} \), \( \Sigma \) by entities in \( \mathcal{R} \). The constants in \( \{ \text{Ur}, \text{Set}, \text{Mom}, \text{Ind}, \text{Chron}, \text{Top}, \text{Subst}, \text{Univ}, \text{RUniv}, \text{Sit} \} \) are interpreted by classes of type \( [0] \), and the constants in \( \{ \in, ::, ;, <, \prec, \subset, \supset, \} \) by classes of \( \text{Typ} [0] \).

Situoid Semantics. Let \( s \) be a situoid and \( \text{Cont}(s) = \{ m \mid m \rhd T \} \); \( \text{Cont}(s) \) is the class of entities contained in \( s \). Note that not every part of an entity in \( s \) is necessarily contained in \( s \), because every situoid has a certain granularity which is determined by the universals \( \text{Gr}(s) = \{ u \mid gr(s, u) \} \). The Ontogram of \( s \) is defined by the triple \( \text{Ont}(s) = (s, \text{Cont}(s), \text{Gr}(s)) \).

Let \( \phi \in L(\text{GOL}) \) be an arbitrary formula of GOL and \( s \) a situoid. A function \( \nu \) is an anchor for \( \phi \) in \( s \) if \( \nu \) associates to every symbol in \( \text{sign}(\phi) \) an element from \( \text{Cont}(s) \cup \text{Gr}(s) \). Here, \( \text{sign}(\phi) \) is the set containing both free variables in \( \phi \) and all the symbols in \( \phi \) denoting contents, universals, classes or sets. The function \( \nu \) has to preserve the type of the symbol. We may then define, using Tarski’s definition of truth, the notion “The anchor \( \nu \) satisfies the sentence \( \phi \) in \( s \)”, which is denoted by \( s \models_\nu \phi \). We may translate certain natural language sentence \( \phi \) into an expression \( \text{tr}(\phi) \) of \( L(\text{GOL}) \) and then we interpret the formula \( \text{tr}(\phi) \) using the sketched semantics for \( L(\text{GOL}) \). We demonstrate this idea by two examples.

**Example 1.** Let us consider the sentence \( \phi = \text{John is kissing Mary} \). The words \( \text{John} \) and \( \text{Mary} \) denote individuals \( j \) and \( m \), \( u_{ke} \) denotes the universal kissing-event. A suitable translation \( \text{tr}(\phi) \) of \( \phi \) to GOL gives the following sentence: \( \exists x (x : u_{ke} \land i(x, (j, m))) \), where the words \( \text{John} \) and \( \text{Mary} \) are replaced by the individual constants \( j \) and \( m \). Then, \( \text{tr}(\phi) \) is satisfiable in a certain situation \( s \) iff there is an anchor \( \nu \) for \( \text{tr}(\phi) \) in \( s \) such that \( s \models_\nu \text{tr}(\phi) \).

**Example 2.** A more developed linguistic theory has to make further distinctions and has to introduce additional ontologically basic relations. Let \( \psi \) be a sentence like “A man kisses a woman twice”. Then we introduce the sentence type \( U(\psi) \) denoting a universal. The instances of \( U(\psi) \) are the individual graphic or acoustic occurrences of the sentence \( \psi \). For example, we can formalize “John says (writes) that \( \psi \)” by \( \exists s (r : U(\text{saying}) \land \text{doing}(J, r) \land s : U(\psi) \land \text{intend}(r, s)) \). Next we consider the thought (thought type) associated with \( \psi \), denoted by \( U(< \psi >) \)
which is a universal whose instances are certain mental events. Thus “it is thought that $\psi$” may be formalized by $\exists (t : U(< \psi >))$. Finally, there is a universal “state of affair” $U(\{\psi\})$ associated with $\psi$. The instances of $U(\{\psi\})$ are individual state of affairs contained in a situoid. “John sees that $\psi$” can be formalized as $\exists u, s(u : U(seeing) \land doing(J,u) \land s : U(\{\psi\}) \land intend(u,s))$. Here, $doing(x,y)$, $intend(x,y)$ are two new ontologically basic relations.

### 6.3 Comparison to other Languages

In this section we compare our approach to that of $KIF$, [16] $E$-logic, [26], and the family of description logics. It turns out that these language are rather weak because they are based on set theory, which is crippled by its extensionalism.

**Knowledge Interchange Format.** Knowledge Interchange Format ($KIF$) is a formal language for the interchange of knowledge among computer programs, written by different programmers, at different times, in different languages. $KIF$ can be considered as a lower-level knowledge modelling language; when a program reads a knowledge base in $KIF$, it converts the data into its own internal form; when the program needs to communicate with another program, it maps its internal data structures into $KIF$.

$KIF$ has the following essential features. The language has a declarative semantics. It is possible to understand the meaning of expressions in the language without appeal to an interpreter for manipulating those expressions. In this way, $KIF$ differs from other languages that are based on specific interpreters, such as Emycin and Prolog. The language provides for the expression of arbitrary sentences in predicate calculus. Hence, it differs from relational database languages (many of which are confined to ground atomic sentences) and Prolog-like languages (that are confined to Horn clauses). The language provides for the representation of knowledge about the representation of knowledge. This allows to make all knowledge representation decisions explicit and permits to introduce new knowledge representation constructs without changing the language.

The ontological basis of $KIF$ can be extracted from [16]; we summarize the main points. The most general ontological entity in $KIF$ is an object. The notion of an object, used in $KIF$, is quite broad; objects can be concrete (e.g., a specific carbon, Nietzsche, the moon) or abstract (the concepts of justice, the number two); objects can be primitive or composite, and even fictional (e.g., a unicorn). In $GOL$, in contrast to $KIF$, there is an ontological classification of the basic entities. $KIF$, a fundamental distinction is drawn only between individuals and sets. A set is a collection of objects; an individual is any object that is not a set. This distinction corresponds in $GOL$ to the difference between urelements and sets. $KIF$ adopts a version of the Neumann-Bernays-Gödel set theory, $GOL$ assumes $ZF$ set theory; but this difference is not essential. The functions and relations in $KIF$ are introduced as sets of finite lists; here the term “set” corresponds to the term “class”. Obviously, the relations and functions in $KIF$ correspond in $GOL$ to the extensional relations. $KIF$ does not provide ontologically basic
relations like our inheritance, part-whole and the like. Hence, the ontological basis of KIF is much weaker than that of GOL. GOL can be considered as a proper extension of KIF; KIF can be understood as the extensional part of GOL.

Frame-Logic. The term “object-oriented approach”, is only a loosely defined, and comprehends a number of notions, such as complex objects, object identity, methods, encapsulation, typing and inheritance, which have been identified as its most important features. One of the main problems with the object-oriented approach is the lack of a logical semantics. Frame-logic is a language that accounts in a declarative fashion for most of the structural aspects of object-oriented and frame-based languages. Furthermore, it is suitable for defining, querying, and manipulating database schemata. F-logic has a model-theoretic semantics and a sound and complete proof theory. F-logic stands in the same relationship to the object-oriented paradigm as classical predicate calculus stands to relational programming. The ontological basis of this language is purely set-theoretical. The instantiation relation, denoted by ;, is modeled by the membership relation, the is-a-relation :: can be explicitly defined in terms of ;. The ontological basis seems to be even weaker than that of KIF, because the full ZF or GB-system is not available. Similarly as for KIF, F-logic captures (some) extensional aspects of GOL.

Description Logic. Description logics are specialized languages related to the KL-ONE system of Brachman and Schmolze [7]. They are designed for representing knowledge, and the general aim is to provide a small set of operations to describe pieces of information, together with efficient methods to make inferences. Description logics are generally considered to be variations of first-order logic - either restrictions or restrictions plus some added operators. These variations are motivated by the undecidability of the inference problem for first order logic and by the intention to preserve the structure of knowledge to be represented.

The ontological basis of description logics is again set theory, in particular the semantics of first-order predicate calculus. But, in addition, as formulated in [27], the language has to be restricted to formulas of a certain form. This philosophy behind this is called in [14] the restricted language thesis. One argument in [27] is that general-purpose knowledge representation systems should restrict their languages by omitting constructs which require non-polynomial (or otherwise unacceptably long) worst-case response time for the correct classification of concepts.

In [14] the position is taken that the restricted language assumptions are flawed. In wider practice the terminological facilities of such systems are so impoverished that the very purpose of general-purpose representational utilities is defeated. In [14] a list of representative examples is expounded that show that many important classes of definable concepts are inexpressible in the restricted languages of KL-ONE and its descendents. These restrictions severely impair the utility for representing and modelling knowledge.
7 Axiomatic Fragments of Mereotopology

In this section we collect several examples to explain, demonstrate and clarify the previous concepts.

Mereotopology and Morphology. There are several approaches to the ontology of space; many of them are based on the conviction that the standard mathematical treatment, using point-set topology, is not sufficient to capture real-world phenomena. Mereology was developed as an alternative to set theory, and the part-whole relation \( \leq \) is one of the ontological basic relations needed to describe the world. But a purely mereological outlook is too narrow to describe spatial entities and the structure of spatial localizations. In Varzi [38] several strategies are expounded to cope with this problem of expressivity; we take here the most obvious approach, namely to add further space-relevant basic relations to the mereological basic relation \( < \). Topology provides a natural next step after mereology in the development of a comprehensive part-whole theory; thus a theory of parts and wholes needs to incorporate a topological machinery of some sort. Such needs will be obvious, especially so in connection with qualitative reasoning about space and time; one needs topology to express the fact that two objects or events are continuously connected, or for the relation of something being inside, or surrounding something else, or being a hole.

Following [5] we distinguish three levels for the description of spatial entities: the mereological level (mereology), the topological level (topology), and the morphological level (morphology). Topology is concerned with such space-relevant properties and relations as connection, coincidence, touching, and continuity. Morphology (also called qualitative geometry) analyses the shape, and relative size of spatial entities. We sketch here some fragments of such theories.

Mereology. In the sequel we use the following definition for reflexive overlap:
\[
\text{ov}(x, y) = \exists z (x \leq z \wedge z \leq y)
\]

1. \( x \leq x \)
2. \( x \leq y \wedge y \leq z \rightarrow x = y \)
3. \( x \leq y \wedge y \leq z \rightarrow x \leq z \)
4. \( \neg x \leq y \rightarrow \exists z (z \leq x \wedge \neg \text{ov}(z, y)) \) (supplementation axiom)
5. \( \exists x \phi(x) \rightarrow \exists z (\forall u (\text{ov}(u, z) \leftrightarrow \exists v (\phi(v) \wedge \text{ov}(u, v))) \)
Topology. There are several possibilities to formalize topological phenomena; here, we will take into consideration the relation of coincidence, based on the ideas of Brentano [8] and the notion of a boundary. The relation of coincidence pertains to boundaries. Let $\equiv$ denote the relation of coincidence and $b(x, y)$ let mean: “$x$ is boundary of $y$”. Let $bd(x) = \exists y(b(x, y)$ and $ip(x, y) = \exists y x \leq y \land \forall z(b(z, y) \rightarrow -occ(z, y))$ (interior parthood). The following axioms are discussed in [32], [35].

1. $x \sim x$
2. $x \sim y \rightarrow y \sim x$
3. $x \sim y \land y \sim z \rightarrow x \sim z$
4. $b(x, y) \rightarrow Top(y)$
5. $b(x, y) \rightarrow x \leq y$
6. $b(x, y) \land b(y, z) \rightarrow b(x, z)$
7. $bd(x) \rightarrow \exists y \exists z(b(x, y) \land Ip(z, y))$

Boundaries can be understood as surfaces of topoids; surfaces can have boundaries which we call lines, and lines can have boundaries called points. We introduce further relations $b_l(x, y)$ (“$x$ is a boundary of the surface $y$”), $b_p(x, y)$ (“$x$ is a boundary of the line $y$”). We introduce the definitions $bd_l(x) = \exists y b_l(x, y)$, and $bd_p(x, y) = \exists y b_p(x, y)$. Then we assume following axioms:

1. $b_l(x, y) \rightarrow bd_l(y)$
2. $b_p(x, y) \rightarrow bd_l(y)$
3. $b_l(x, y) \rightarrow x \leq y$
4. $b_p(x, y) \rightarrow x \leq y$
5. $x \sim y \rightarrow (bd_l(x) \land bd_l(y)) \lor (bd_l(x) \land bd_l(y)) \lor (bd_p(x) \land bd_p(y))$

The (strong) connectedness of a topoid may be defined as follows. $C(x) = \forall y z(x = y \sqcup z \land Top(y) \land Top(z) \rightarrow \exists s(r \leq y \land b(r, y) \land b(s, z) \land s \leq z \land r \sim s))$. Analogously, the connectedness of boundaries is defined by $C_s(x) = \forall y z(x = y \sqcup z \land bd_l(y) \land bd_l(z) \rightarrow \exists s(r \leq y \land s \leq z \land r \sim s))$. $C_l(x)$ denotes the connectedness predicate for lines (as boundaries of surfaces). Using the relations $\mu$, $b(x, y)$, $b_l(x, y)$, $b_p(x, y)$ and $\sim$ a mereotopological theory of pure topoids (topoids separated from other categories of individuals) could be developed.

If we want to integrate other categories of individuals, say substances, in this theory then certain complications arise, and it seems that a fully developed mereotopological theory including all kinds of individuals does not yet exist. We discuss some of these problems for the case of substances. The world of pure topoids and the world of substances are connected by a basic relation $occ(x, y)$ having the intuitive meaning “the individual $x$ occupies the topoid $y$”.

It seems to be that the relation $occ(x, y)$ is assumed to draw flat boundaries. A flat boundary (excluding cognitive decision) through a substance $s$ could be defined as a suitable mereological partition of $s$ which is derived from a prior partition of the pure topoid occupied by $s$.
The following axiom seems to be plausible:

\[ \forall x (\text{Subst}(x) \rightarrow \exists t (\text{Top}(t) \land \text{occ}(x, t))). \]

Substances may have boundaries, but the boundaries of substances are not the same as the boundaries of the topoids they occupy. Thus, we introduce a new boundary-relation \( bs(x, y) \) having the meaning “\( x \) is a substantial boundary of the substance \( y \)”. The following axiom might then be acceptable:

\[ \forall xyz (\text{Subst}(x) \land \text{occ}(x, t) \land bs(y, x) \land b(z, t) \rightarrow z \sim y) \]

The connectedness of a substance can be defined as follows. \( Cn(x) =_d \neg \exists t (\text{occ}(x, t) \rightarrow \neg \forall t (C(t))) \), i.e., every topoid occupied by \( x \) is connected. This definition can be extended to the connectedness of a boundary. In contradistinction to \[30\] we admit substances without connected boundaries. The following axiom says that every connected substance has a substantial boundary which is itself connected.

\[ \forall x (\text{Subst}(x) \land Cn(x) \rightarrow \exists y (bs(y, x) \land Cn(y))). \]

**Morphology.** To describe the form of an object in \[5\] we adopt a relation of *congruence*, denoted by \( \cong \), between regions whose intended meaning is: “two regions are congruent if they have the same shape and size”. For every topoid \( t \) one could introduce a universal \( U_t \) whose instances are topoids that are congruent with \( t \). This would lead to a theory of shapes for pure topoids, separated from a theory substances. The universals \( U_t \) are sub-universals of more general ones; we are interested in particular in the universal \( U_{sp} \) whose instances are spheres and \( U_{sh} \) whose instances are cubes. A minimal sub-universal of \( U_{sp} \) has all spheres with the same radius as its instances.

Again, there is the problem how to connect the morphological theory of pure topoids to the world of substances. Our approach is the following. There is a universal \( U_{sh} \) whose instances are shapes; shapes are understood as moments, i.e., we have the axiom \( \forall x (x : U_{sh} \rightarrow \text{Mom}(x)) \). Obviously, if a substance has a shape then it has a boundary. Hence, we admit the following axiom

\[ \forall xy (\text{Subst}(x) \land y : U_{sh} \land i(y, x) \rightarrow \exists z (bs(z, x))) \]

Shapes are transmitted from substances to the topoids they occupy; we say that a topoid \( t \) has a shape related to \( s \) if there is substance with shape \( s \) occupying \( t \). Assume there is a substance \( s \) with shape \( a \), and \( s \) moves from a position \( p_1 \) to a position \( p_2 \), and assume that \( a \) is invariant during this movement, then the topoids occupied by \( s \) in positions \( p_1 \) and \( p_2 \) are congruent.\(^3\)

8 **Acknowledgements**

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\(^3\) To make this precise one has to understand what it means that a moment is invariant during a time period.
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