Representing Meta-Knowledge in Nonmonotonic Inference Systems

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1. Motivating example

Peter tells you that your girl-friend Anne went out for dinner with another man yesterday. Peter even knows his name: it was John, a highly attractive person known for having numerous affairs.

You are concerned and talk to Anne about this. She tells you she was at home yesterday evening waiting for you to call.

Peter insists that he saw Anne with that man. You are not sure what to believe. Luckily, you find out that Anne has a twin sister Mary. Mary indeed went out with her new boy-friend John.

This explains why Peter got mixed up. You now believe Anne and happily continue your relationship.

2. Underlying ideas

- drawing conclusions based on defeasible information involves reasoning about meta-information
- to model such reasoning it should be possible to express and reason about meta-information, including the reliability of formulas, in the logic
- opens possibility to represent conflict handling strategies declaratively

Poole systems (F, D): a reminder
Consistent set of formulas F (facts), possibly inconsistent set of formulas D (defaults). E is an extension of (F, D) iff $E = Th(F \cup D')$ where D' is a maximal F-consistent subset of D.

p provable iff p is contained in all extensions.

Example

$$D = \{bird(x) \rightarrow flies(x)\}$$
$$F = \{\forall x.\, penguin(x) \rightarrow \neg flies(x),$$
$$bird(tweety), penguin(tweety), bird(hansi)\}$$

Single extension contains F and instances of bird rule with $x \neq tweety$.

$flies(hansi)$ and $\neg flies(tweety)$ provable

Here for simplicity F assumed empty, D finite: every piece of information defeasible

Formalization

Definition 1. A named formula is a structure of the form $d{:}p$, where p is a first order formula and d a ground term representing the name of the formula.

Definition 2. A preference default theory T is a finite set of named formulas such that

- $form(T)$ is a set of first order formulas whose logical language contains a reserved symbol $<$ representing a strict partial order, and
- $d_1{:}p \in T$, $d_2{:}q \in T$ and $p \neq q$ implies $d_1 \neq d_2$.

Definition 3. Let T be a preference default theory, $\prec$ a total order on T. The extension of T generated by $\prec$, denoted $E_T^{\prec}$, is the set $E_T^{\prec} = Th(\bigcup_{i=0}^{|T|} E_i)$ where

- $E_0 = \emptyset$, and for $0 < i \leq |T|$
- $E_i = E_{i-1} \cup \{form(d_i)\}$ if this set is consistent, E_{i-1} otherwise. Here d_i is the i-th element of T according to the total order $\prec$.

Definition 4. Let T be a preference default theory, $\prec$ a strict partial order on T. The set of extensions of T generated by $\prec$ is

$$Ext_T^{\prec} = \{E_T^{\prec'} \mid \prec' \text{ is a total order extending } \prec\}.$$

Definition 5. Let T be a preference default theory, $\prec$ a strict partial ordering of T, S a set of formulas. We say $\prec$ is compatible with S iff

$$S \cup \{d < d' \mid d{:}p \prec d'{:}q\} \cup \{\neg(d < d') \mid d{:}p \not\prec d'{:}q\}$$

is consistent.

An extension E of T is compatible with S iff there is a strict partial ordering $\prec$ of T compatible with S such that $E \in Ext_T^{\prec}$.

The set of extensions of T compatible with S is denoted Ext_T^S.

Definition 6. Let T be a preference default theory. A set of formulas E is called a preferred extension of T iff E is compatible with itself, that is, $E \in Ext_T^E$.

Examples

$d_1(x) : bird(x) \to flies(x) \mid x$ ground object term

$d_2 : \forall x.\ penguin(x) \to \neg flies(x)$

$d_3 : bird(tweety) \wedge penguin(tweety)$

$d_4 : \forall x.\ d_3 < d_1(x)$

$d_5 : \forall x.\ d_2 < d_1(x)$

3 extensions, single preferred extension contains $\neg flies(tweety)$

Problem: non-existence of extensions

$$d_1 : d_2 < d_1$$
$$d_2 : d_1 < d_2$$

Not all sets of formulas represent reasonable belief sets under this semantics

Guaranteeing consistent belief sets

Definition 7. Let T be a preference default theory, S a set of formulas. We define an operator C_T as follows:

$$C_T(S) = \bigcap Ext_T^S$$

Proposition 8. *The operator C_T is monotone.*

Definition 9. Let T be a preference default theory. A formula p is an accepted conclusion of T iff $p \in lfp(C_T)$, where $lfp(C_T)$ is the least fixed point of the operator C_T.

We call extensions which are compatible with $lfp(C_T)$ accepted extensions.

Proposition 10. *Let T be a preference default theory, p an accepted conclusion of T. Then p is contained in all preferred extensions of T.*

Proposition 11. *Let T be a preference default theory. The set of accepted conclusions of T is consistent.*

Example 1

$d_1 : penguin(tweety)$

$d_2 : \forall x.penguin(x) \rightarrow bird(x)$

$d_3 : \forall x.penguin(x) \rightarrow \neg flies(x)$

$d_4(x) : bird(x) \rightarrow flies(x) \mid x$ ground object term

$d_5 : observation(d_1)$

$d_6 : rule(d_2)$

$d_7 : rule(d_3)$

$d_8 : \forall x.\ default(d_4(x))$

$d_9 : \forall n, n'.\ rule(n) \wedge observation(n') \rightarrow n < n'$

$d_{10} : \forall n, n'.\ observation(n) \wedge default(n') \rightarrow n < n'$

Example 2

$d_1 : p$

$d_2 : \neg p$

$d_3 : time(d_1) = 10$

$d_4 : time(d_2) = 11$

$d_5 : source(d_1) = Peter$

$d_6 : source(d_2) = John$

$d_7 : more\text{-}rel(Peter, John)$

$$d_8(n, n') : more\text{-}rel(source(n), source(n')) \rightarrow n < n' \quad | \; n, n' \in \{d_1, \ldots, d_7\}$$

$$d_9(n, n') : time(n) < time(n') \rightarrow n' < n \quad | \; n, n' \in \{d_1, \ldots, d_7\}$$

$d_{10} : \forall n, n'. \; \{d_3, \ldots, d_7\} < d_8(n, n') < d_9(n, n')$

Example 3

$d_1 : sunday \to visit\text{-}mom$

$d_2 : puccini \to go\text{-}opera$

$d_3 : sunday \wedge puccini \ (\wedge \ birthday)$

$d_4 : mom's\text{-}rule(d_1)$

$d_5 : wife's\text{-}rule(d_2)$

$d_6(x, y) : mom's\text{-}rule(x) \wedge wife's\text{-}rule(y) \to x < y$

$d_7(x, y) : birthday \wedge mom's\text{-}rule(x) \wedge wife's\text{-}rule(y) \to y < x$

$d_8 : \{d_3, d_4, d_5\} < \{d_1, d_2\} \wedge \forall x, y.\ d_7(x, y) < d_6(x, y)$

Example 4

$d_1 : date(Anne, John)$

$d_2 : \neg date(Anne, John)$

$d_3 : rel(d_1) = medium$

$d_4 : rel(d_2) = medium$

$d_5 : date(Mary, John)$

$d_6 : twins(Mary, Anne)$

$d_7 : date(Mary, John) \land twins(Mary, Anne) \to rel(d_1) = low$

$d_8 : \forall n, n'.\, rel(n) = high \land rel(n') = medium \to n < n'$

$d_9 : \forall n, n'.\, rel(n) = medium \land rel(n') = low \to n < n'$

$d_{10} : rel(d_5) = rel(d_6) = rel(d_7) = rel(d_8) = rel(d_9) = high$

$d_{11} : rel(d_3) = rel(d_4) = medium$

Concluding remarks

- Representing and reasoning with meta-information essential for commonsense reasoning.
- Addition of names for formulas and standard predicate $<$ expressing reliability or preference necessary.
- Preferred extensions are compatible with themselves. Existence not guaranteed.
- Accepted extensions are compatible with intersection of all accepted extensions. Existence guaranteed.
- Framework can be used to model (trivialize) iterated belief revision.
- A methodology for representing information in this framework?