A Rank Based Language for Qualitative Preferences

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Outline

1. Motivation
2. Ranked KBs for specifying preferences
3. Complex preferences: the language LPD
4. Example: selecting a movie
5. Conclusions
Problems and solutions

- problem solutions often value assignments: for each variable $v$ pick a value $a$ out of domain $D(v)$
- constraints describe legal assignments (here: boolean variables and set of formulas $B$, solutions models of $B$)
- legal assignments not necessarily of same quality => preferences
- given the number of assignments is exponential: how to describe the preferences?
Goal based preferences

simplest case: single formula $G$ representing goal

- $m_1$ better than $m_2$ iff $m_1 \models G, m_2 \not\models G$

agents have more than 1 goal
goals are of different importance

- use ranked knowledge base

$$K = \{(G_1, r_1), \ldots, (G_n, r_n)\}$$

$r_i$ integers, $G_i$ more important than $G_j$ iff $r_i > r_j$

- focus on qualitative approaches where numbers only used to represent total preorder on goals
Preferences on models

<table>
<thead>
<tr>
<th>rank</th>
<th>$m_1$</th>
<th>$m_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td>$G_4, G_5, G_6$</td>
<td>$G_4, G_5, G_6$</td>
</tr>
<tr>
<td>2</td>
<td>$G_2, G_3$</td>
<td>$G_2, G_3$</td>
</tr>
<tr>
<td>1</td>
<td>$G_1$</td>
<td>$G'_1$</td>
</tr>
</tbody>
</table>

Is $m_1$ better than $m_2$?

- No if importance of best satisfied goal counts
- No if importance of best falsified goal counts
- Yes if number of satisfied goals at highest level where they differ counts
- No if subset relation at highest level where they differ counts
Basic strategies

\( m_1 \) better than \( m_2 \) iff

\[ \top \text{ rank of highest goal satisfied in } m_1 > \text{ rank of highest goal satisfied in } m_2 \] (Benferhat et al. 02)

\[ \kappa \text{ rank of highest goal unsatisfied in } m_1 < \text{ rank of highest goal unsatisfied in } m_2 \] (Pearl 90)

\[ \# \text{ at highest rank where } m_1 \text{ and } m_2 \text{ differ wrt. number of goals, } m_1 \text{ satisfies more goals} \] (Benferhat et al. 93)

\[ \subseteq \text{ at highest rank where } m_1 \text{ and } m_2 \text{ differ wrt. goals, } m_1 \text{ satisfies superset of goals} \] (Brewka 89)
### Examples

<table>
<thead>
<tr>
<th>$m_1$</th>
<th>$m_2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$G_4, G_5, G_6$</td>
<td></td>
</tr>
<tr>
<td>$G_2, G_3$</td>
<td></td>
</tr>
<tr>
<td>$G_1$</td>
<td>$G_4, G_5, G_6$</td>
</tr>
<tr>
<td>$G_2, G_3$</td>
<td></td>
</tr>
<tr>
<td>$G_1$</td>
<td></td>
</tr>
</tbody>
</table>

$\geq_\kappa, =\top$

$=\kappa, >\top$
Relationships

\[ m_1 \succ K \quad m_2 \implies m_1 \succ K m_2, \]
\[ m_1 \succ K \quad m_2 \implies m_1 \succ K m_2, \]
\[ m_1 \succ K \quad m_2 \implies m_1 \succ \# m_2, \]
\[ m_1 \succ K \quad m_2 \implies m_1 \succ \# m_2, \]
\[ m_1 \succ K \quad m_2 \implies m_1 \succeq K m_2, \]
\[ m_1 \succ K \quad m_2 \implies m_1 \succeq K m_2. \]
The preference language

Basic preference expressions:

- pair \((s, K)\), \(s \in \{\top, \kappa, \subseteq, \#\}\) strategy,
  \(K\) ranked goal base, written \(K^s\)

single agent: different strategies for different aspects
multiple agents: generate global preference ordering
  \(\Rightarrow\) LPD:

- basic preference expressions are in LPD
- if \(d_1\) and \(d_2\) are in LPD, so are \((d_1 \land d_2)\),
  \((d_1 \lor d_2)\), \((d_1 > d_2)\) and \(-d_1\).
Semantics of LPD

- meaning of expressions: preorder $\geq$ on models $m_1 > m_2$ iff $m_1 \geq m_2$ but $m_2 \not\geq m_1$
- basic expressions induce preorder on models as discussed earlier
- $R_1$ and $R_2$ preorders induced by $d_1$ and $d_2$, $R^{-1}$ inverse of $R$. The preorders induced by a complex expression are:
  
  $d_1 \land d_2 \Rightarrow R_1 \cap R_2$
  $d_1 \lor d_2 \Rightarrow tr(R_1 \cup R_2)$
  $d_1 > d_2 \Rightarrow (R_1 \cap R_2) \cup (R_1 \setminus R_1^{-1})$
  $-d_1 \Rightarrow R_1^{-1}$
The combination methods

∧ Pareto ordering: \( m_1 \) at least as good as \( m_2 \) iff at least as good with respect to \( R_1 \) and \( R_2 \)

∨ \( m_1 \) at least as good as \( m_2 \) iff at least as good with respect to \( R_1 \) or \( R_2 \), needs transitive closure

> lexicographic ordering: \( m_1 \) at least as good as \( m_2 \) iff at least as good with respect to \( R_1 \) and \( R_2 \), or strictly better wrt. \( R_1 \)

– inverse ordering
Example: selecting a movie

\[ K_1 : \{(Hugh, 2), (Brad, 2), (Leo, 1)\} \]
\[ K_2 : \{(Julia, 2), (Nicole, 2), (Gwyneth, 1), (Halle, 1)\} \]
\[ K_3 : \{(comedy, 3), (action, 2), (tragedy, 1)\} \]

- \( K_1 \) wife’s actors preferences, \( K_2 \) yours,
- \( K_3 \) type preferences,
- \( K_1 \) more important than \( K_2 \) (wife’s birthday),
  type as important as actors.

Strategy:

\[ (K_1^\# > K_2^\#) \land K_3^\top \]
Generating the models

\[ M_1 : \text{comedy}, Hugh, Brad \]
\[ M_2 : \text{comedy}, Hugh, Leo, Julia \]
\[ M_3 : \text{comedy}, Brad, Leo, Julia, Halle \]
\[ M_4 : \text{action}, Brad, Hugh, Nicole \]
\[ M_5 : \text{action}, Brad, Leo, Julia, Halle \]
\[ M_6 : \text{tragedy}, Brad, Leo, Julia, Nicole \]

complete movie information, represented in \( B \) as

\[ M_1 \rightarrow \text{comedy} \]
\[ \land Hugh \land Brad \land \neg Leo \]
\[ \land \neg Julia \land \neg Nicole \land \neg Gwyneth \land \neg Halle \]
Preferences among movies

- types exclusive: "comedy $\rightarrow \neg action$, ... 
- select 1 movie: $M_1 \lor \ldots \lor M_6$, $M_1 \rightarrow \neg M_2$, ...
- info in background knowledge $B$: each model makes 1 movie true

Diagram:

$M_1$  $\rightarrow$  $M_2$  $\rightarrow$  $M_3$  $\leftarrow$  $M_5$  $\rightarrow$  $M_4$  $\rightarrow$  $M_6$
Why is this nonmon?

- call for papers explicitly mentions preferences
- preferred solutions change when background knowledge or goals added
- NMR via preference on models standard approach
- for $d \in LPD$ define $p \models_d q$ iff $q$ holds in all maximally $d$-preferred models of $p$
Discussion

- related paper at KR: based on rules rather than goals, handles default negation, focus on answer set optimization
- CP-networks: ceteris paribus interpretation of preferences; here: a multi criteria view
- Benferhat et al.: bipolar representations: goals and rejections, rejections can be modeled using negation and adequate strategy
- a lot more related work -> see paper
- Future work: properties of the language, partially ordered goals, computational issues