Handling Exceptions in Knowledge Representation: A Brief Introduction to Nonmonotonic Reasoning

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Part I: Background and Simple Forms of Nonmon

1. Background and Motivation
2. Closed World Assumption
3. Argumentation Frameworks
Part I: Background and Simple Forms of Nonmon

1 Background and Motivation
2 Closed World Assumption
3 Argumentation Frameworks

Part II: Answer Set Programming
Part I

Background and Simple Forms of Nonmon
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  \[ \forall x. \text{PhDstudent}(x) \rightarrow \text{Student}(x) \]

- Useful, e.g. for concept definitions or in mathematics
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- Birds fly ...

- Owls hunt at night ...

- Students hate theoretical computer science ...

- After spending 2 hours in the doctor's waiting room patients get angry ...
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  - Students hate theoretical computer science ... unless they are very clever.
  - After spending 2 hours in the doctor’s waiting room patients get angry ... unless they are close to finishing a proof.
  - ...
A solution?

- Most of our commonsense knowledge is of this kind
- What can we do to represent it adequately?
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- What can we do to represent it adequately?
- What if instead of $\forall x. \text{Bird}(x) \rightarrow \text{Flies}(x)$ we use
  $$\forall x. \text{Bird}(x) \land \neg \text{Ab}(x) \rightarrow \text{Flies}(x)$$
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- Problem 1: no exhaustive list of abnormalities.
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- Problem 1: no exhaustive list of abnormalities.
- Problem 2: does not give us $Flies(tweety)$ unless $tweety$ is known not to be an exception.
How to use generic information

- Want to draw conclusions from generic information *as long as nothing indicates an exception*.
- If additional information tells us something is abnormal, retract former conclusion.
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  ⇒ *Conclusions do not grow monotonically with premises.*
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  \[ \Rightarrow \text{Conclusions do not grow monotonically with premises.} \]

- Classical logic cannot model this, as it is monotonic:
  \[ X \subseteq Y \Rightarrow \text{Th}(X) \subseteq \text{Th}(Y). \]

- Why? \( q \) follows from \( X \) if \( q \) holds in all models of \( X \). Models of \( Y \) a subset, thus \( q \) holds in all of them as well.
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  Why? \( q \) follows from \( X \) if \( q \) holds in all models of \( X \). Models of \( Y \) a subset, thus \( q \) holds in all of them as well.
- Observation led to the AI field of nonmonotonic reasoning, active for over 30 years.
Defaults may give rise to conflicting conclusions:

1. Quakers normally are pacifists.
2. Republicans normally are not pacifists.
3. Nixon is a quaker and a republican.

(1) and (2) conflicting.

Nothing wrong with the defaults!
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(3) *Nixon is a quaker and a republican.*

(1) and (2) conflicting.

Nothing wrong with the defaults!

Different approaches to deal with this:

- some apply none of the conflicting defaults,
- most generate different acceptable belief sets (extensions) leave open whether to use them sceptically (*p* true in all of them) or credulously (*p* true in some of them, or in a particular one).
Check the QuantLA Spring School time table

*Question:* *Is Franz teaching on Friday?*

*Your answer (presumably): No*
2. The Closed World Assumption

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- Why is this answer correct?

- Does not follow from the explicit information in the time table
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- Check the QuantLA Spring School time table
  - Question: Is Franz teaching on Friday?
  - Your answer (presumably): No

Why is this answer correct?

- Does not follow from the explicit information in the time table
- But: follows from this information assuming that the list of courses is complete

You (presumably) used this assumption, and do so in many everyday contexts
The Closed World Assumption, ctd.

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- Communication convention: represent the latter only, leave the former implicit.
  - train/flight schedules
  - TV programs
  - library catalogues
  - list of lectures at a spring school
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- Communication convention: represent the latter only, leave the former implicit.
  - train/flight schedules
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  - list of lectures at a spring school

- Know how to infer negative information based on completeness assumption.
Reiter’s formalization

Let $KB$ be a set of formulas, define new form of entailment under CWA:

$$KB \models_c \alpha \iff KB \cup Negs \models \alpha$$

where $Negs = \{\neg p \mid p \text{ atomic and } KB \not\models p\}$
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- CWA makes knowledge complete: for arbitrary $\alpha$ (without quantifiers) we have $KB \models_c \alpha$ or $KB \models_c \neg \alpha$.

- Recursive query evaluation; queries reduced to atomic case.
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- CWA makes knowledge complete: for arbitrary $\alpha$ (without quantifiers) we have $KB \models_c \alpha$ or $KB \models_c \neg \alpha$.

- Recursive query evaluation; queries reduced to atomic case.

- Results extend to quantified formulas if we add *domain closure assumption* (each object named by constant) and *unique names assumption* (different constants denote different objects).
A major problem

- Works for simple cases only, e.g. KB a set of atoms.
- Assume $KB \models (p \lor q)$, but $KB \not\models p$ and $KB \not\models q$.
- Now $\neg p \in Negs$ and $\neg q \in Negs$, thus $KB \cup Negs$ inconsistent.
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- Weaker versions of CWA avoiding inconsistency were proposed.
- CWA best viewed as a method for restricted contexts (e.g. databases).
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Standard Reference:

2. Argumentation

- Argumentation highly active area in AI.
- Idea: to construct acceptable set(s) of beliefs from given KB:
  1. construct arguments (beliefs with associated reasons),
  2. determine jointly acceptable arguments (extensions),
  3. accept their conclusions.
- Assumption: step 2 can be done independently and abstractly.
- Dung’s Abstract Argumentation Frameworks widely used tool.
Abstract Argumentation

- Arguments “atomic”, their structure irrelevant.
- All that matters are attacks among arguments.
- Argumentation frameworks (AFs) represent attack relations.
- Semantics formalize different intuitions about how to solve conflicts and how to pick acceptable arguments.
- Semantics map an AF to subsets of its arguments (extensions).
- Nonmonotonic: new argument may throw out what was accepted.
Argumentation Frameworks

An argumentation framework (AF) is a pair \((A, R)\) where

- \(A\) is a set of arguments,
- \(R \subseteq A \times A\) is a relation representing “attacks”. (“defeats”)
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**Example**

\[a \rightarrow b \rightarrow c \rightarrow d \rightarrow e\]
Semantics: minimal requirement no conflicts

**Conflict-Free Set**

Given an AF $F = (A, R)$. A set $S \subseteq A$ is **conflict-free** in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

**Example**

$cf(F) = \{\{a, c\}\}$,
Semantics: minimal requirement no conflicts

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Example

\[
\text{cf}(F) = \{ \{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset \}
\]
No undefended attacked arguments

Admissible Set

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$,
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$adm(F) = \{\{a, c\}\}$,
Admissible Set

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$, where $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

![Graph with nodes a, b, c, d, e and edges: a→b, c→d, e→c, d→e, b→c, c→a]

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Example

$$adm(F) = \{ \{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset \}$$
Want all defended arguments

Complete Set
Given an AF $F = (A, R)$. A set $S \subseteq A$ is complete in $F$, if
- $S$ is admissible in $F$
- each $a \in A$ defended by $S$ in $F$ is contained in $S$
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

$comp(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$
An inherently skeptical approach

Grounded Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is grounded in $F$, if

- $S$ is complete in $F$
- for each $T \subseteq A$ complete in $F$, $T \not\subset S$

Proposition [Dung 95]: The grounded extension of an AF $F = (A, R)$ is given by the least fix-point of the operator $\Gamma_F : 2^A \rightarrow 2^A$, defined as $\Gamma_F(S) = \{ a \in A \mid a \text{ is defended by } S \text{ in } F \}$

Example

$ground(F) = \{ \{a, c\}, \{a, d\}, \{a\} \}$
A credulous approach

Stable Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is **stable** in $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$.

Example

$$\begin{align*}
&\begin{array}{c}
  a \\
  b \\
  c \\
  d \\
  e 
  \end{array} \\
&\begin{array}{c}
  \rightarrow \\
  \leftarrow \\
  \leftarrow \\
  \rightarrow \\
  \rightarrow
  \end{array}
\end{align*}$$

$stable(F) = \{a, e\}$,
A credulous approach

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Example

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Nonmonotonic Reasoning
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$$stable(F) = \{\{a, e\}, \{a, d\}\},$$
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Stable($F$) = \{\{a, c\}, \{a, d\}, \{b, d\}\}.
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Example

$$\text{stable}(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$$
Guaranteeing existence of extensions

Preferred Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is preferred in $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $T$, $S \not\subseteq T$

Example

![Graph]

$\text{pref}(F) = \{\{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset\}$
Complexity

Relation between Semantics

- stable
- pref
- compl
- adm
- ground

Dimopoulos & Torres 96; Dunne & Bench-Capon 02; Coste-Marquis et al. 05

Nonmonotonic Reasoning
Complexity

Relation between Semantics

stable ➔ pref ➔ compl ➔ adm

ground

Complexity

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</tbody>
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[Dimopoulos & Torres 96; Dunne & Bench-Capon 02; Coste-Marquis et al. 05]
Further remarks

- AFs: simple graph representation of argumentation scenarios.
- Various semantics model different intuitions how to select reasonable argument sets.

BUT

- Fixed meaning of links: attack; fixed acceptance condition for args: no parent accepted.
- Want more flexibility:
  - Links supporting arguments/positions,
  - Nodes not accepted unless supported,
  - Flexible means of combining attack and support.
- Developed *Dialectical Frameworks* which can have arbitrary relations among args.
- Many options for adding quantities.