A Brief Introduction to Nonmonotonic Reasoning

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Outline

Lecture I: Background and Simple Forms of Nonmon

1. Background and Motivation
2. Closed World Assumption
3. Argumentation Frameworks

Lecture II: The Big Three and ASP

4. Preferences Among Formulas: Poole and Beyond
5. Preferences Among Models: Circumscription
6. Nonstandard Inference Rules: Default Logic
7. Answer Set Programming
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Background and Simple Forms of Nonmon
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- Classical logic allows us to represent universal statements:

\[ \forall x. \text{PhDStudent}(x) \rightarrow \text{Student}(x) \]

- Useful, e.g. for concept definitions or in mathematics

- Less useful to represent generic statements which may have exceptions:
  - Professors teach ... unless they are on sabbatical.
  - Birds fly ... unless they are penguins.
  - Owls hunt at night ... unless they live in a zoo.
  - Students hate theoretical computer science ... unless they are very clever.
  - After spending 2 hours in the doctor’s waiting room patients get angry ... unless they are close to finishing a proof.
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A solution?

- Most of our commonsense knowledge is of this kind
- What can we do to represent it adequately?
  - What if instead of $\forall x. \text{Bird}(x) \rightarrow \text{Flies}(x)$ we use
    $$\forall x. \text{Bird}(x) \land \neg \text{Ab}(x) \rightarrow \text{Flies}(x)$$
    and add
    $$\forall x. \text{Ab}(x) \leftrightarrow \text{Penguin}(x) \lor \text{Ostrich}(x) \lor \text{Injured}(x) \lor \ldots$$
  - Problem 1: no exhaustive list of abnormalities.
  - Problem 2: does not give us $\text{Flies}(\text{tweety})$ unless $\text{tweety}$ is known not to be an exception.
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How to use generic information

• Want to draw conclusions from generic information *as long as nothing indicates an exception.*

• If additional information tells us something is abnormal, retract former conclusion.
  
  ⇒ *Conclusions do not grow monotonically with premises.*

• Classical logic cannot model this, as it is monotonic:

  \[ X \subseteq Y \Rightarrow Th(X) \subseteq Th(Y). \]

• Why? \(q\) follows from \(X\) if \(q\) holds in all models of \(X\). Models of \(Y\) a subset, thus \(q\) holds in all of them as well.

• Observation led to the AI field of nonmonotonic reasoning, active for over 30 years.
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Conflicting Defaults

Defaults may give rise to conflicting conclusions:

(1) Quakers normally are pacifists.
(2) Republicans normally are not pacifists.
(3) Nixon is a quaker and a republican.

(1) and (2) conflicting.

Nothing wrong with the defaults!

Different approaches to deal with this:

• some apply none of the conflicting defaults,
• most generate different acceptable belief sets (extensions)
  leave open whether to use them sceptically (p true in all of them)
  or credulously (p true in some of them, or in a particular one).
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2. The Closed World Assumption

- Check the course time table
  
  - **Question**: Is the course on Knowledge Representation on Friday?
  
  - **Your answer (presumably)**: No

- Why is this answer correct?

- Does not follow from the explicit information in the time table

- But: follows from this information *assuming that the list of courses is complete*

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  • You (presumably) used this assumption, and do so in many everyday contexts
• In many situations way more negative than positive facts.

• Communication convention: represent the latter only, leave the former implicit.
  • train/flight schedules
  • TV programs
  • library catalogues
  • list of lectures

• Know how to infer negative information based on completeness assumption.
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Reiter’s formalization

• Let $KB$ be a set of formulas, define new form of entailment under CWA:

\[ KB \models_c \alpha \iff KB \cup Negs \models \alpha \]

where $Negs = \{\neg p \mid p \text{ atomic and } KB \not\models p\}$

• $\models_c$ nonmonotonic, for instance $\{a\} \models_c \neg b$ whereas $\{a, b\} \not\models_c \neg b$

• CWA makes knowledge complete: for arbitrary $\alpha$ (without quantifiers) we have $KB \models_c \alpha$ or $KB \models_c \neg \alpha$.

• Recursive query evaluation; queries reduced to atomic case.

• Results extend to quantified formulas if we add domain closure assumption (each object named by constant) and unique names assumption (different constants denote different objects).
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A major problem

- Works for simple cases only, e.g. KB a set of atoms.
- Assume $KB \models (p \lor q)$, but $KB \not\models p$ and $KB \not\models q$.
- CWA best viewed as a method for restricted contexts (e.g. databases).

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Weaker versions of CWA

- Generalized CWA (Minker, 1982):

  \[ \text{Negs} = \{ \neg p \mid p \text{ atomic and for every positive clause } C \text{ with } KB \notmodels C, KB \notmodels C \lor p \} \]

- Extended Generalized CWA (Yahya and Henschen, 1985):

  \[ \text{Negs} = \{ \neg K \mid K \text{ a conjunction of atoms and for every positive clause } C \text{ with } KB \notmodels C, KB \notmodels C \lor K \} \]

- Further refinements partition atoms into different groups (Careful CWA, Extended CWA). Extended CWA is equivalent to circumscription for propositional logic.
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The Big Three and ASP
4. Preferences Among Formulas: Poole and Beyond

- Treat defaults as classical formulas with lower priority.
- Partition KB into (consistent) strict part $F$ and defeasible part $W$.
- In case of a conflict give up formulas from the latter set, that is consider “scenarios” (Poole) of the form

$$F \cup W'$$

where $W'$ is a maximal $F$-consistent subset of $W$.

**Example**

$F = \{ \text{bird(tweety)}, \text{bird(fritz)}, \neg \text{flies(fritz)} \}$

$W = \{ \text{bird(tweety)} \rightarrow \text{flies(tweety)}, \text{bird(fritz)} \rightarrow \text{flies(fritz)} \}$

Scenario: $F \cup \{ \text{bird(tweety)} \rightarrow \text{flies(tweety)} \}$

Conclude $\text{flies(tweety)}$ from single scenario.
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Conclude $\text{flies(tweety)}$ from single scenario.
May get multiple scenarios.

Skeptical vs. credulous reasoning: \( p \) follows from all scenarios vs. \( p \) follows from some scenario.

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Scenario 1: \( F \cup \{ \text{bird(tweety)} \rightarrow \text{flies(tweety)} \} \)

Scenario 2: \( F \cup \{ \text{peng(tweety)} \rightarrow \neg \text{flies(tweety)} \} \)

Neither \( \text{flies(tweety)} \) nor \( \neg \text{flies(tweety)} \) follows skeptically.

Important to represent instances of *Birds fly*, not universal formula (otherwise single nonflying bird eliminates the default).

Example suggests generalization: defaults preferred to others.
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Example

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Preferred subtheories

- Basic idea: introduce arbitrary preference levels.
- Rather than \((F, W)\) use partition \(KB = (F_1, \ldots, F_n)\); \(F_1\) most reliable formulas, \(F_2\) second best, etc.
- Preferred subtheory: maxi-consistent subset \(S\) of \(F_1 \cup \ldots \cup F_n\) containing maxi-consistent subset of \(F_1 \cup \ldots \cup F_i\) for each \(i \leq n\).
- Intuition: pick maxi-consistent subset of \(F_1\), extend it maximally with formulas from \(F_2\), etc.
Example

\[ F_1 = \{ \text{bird(tweety)}, \text{penguin(tweety)} \} \]

\[ F_2 = \{ \text{penguin(tweety)} \rightarrow \neg \text{flies(tweety)} \} \]

\[ F_3 = \{ \text{bird(tweety)} \rightarrow \text{flies(tweety)} \} \]

Single preferred subtheory: \( F_1 \cup F_2 \)

\( \neg \text{flies(tweety)} \) follows skeptically
Remarks

- Simple approach reducing default reasoning to inconsistency handling.
- No nonstandard semantics, no nonstandard language constructs.
- Easy handling of preferences.
- Quantitative extensions straightforward, e.g. reliability value for each formula, consistent subsets ranked by sum of values.
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Let’s do this for selected predicates/atoms only.

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Comes with a default representation scheme (ab predicates):

\[ \forall x. Bird(x) \land \neg Ab(x) \rightarrow Flies(x). \]

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Example

$KB = \{ bird, bird \land \neg ab \rightarrow flies \}$

Models:

$M_1 = \{ bird, ab, flies \}, \ M_2 = \{ bird, ab, \neg flies \}, \ M_3 = \{ bird, \neg ab, flies \}$

- $M_1$ and $M_2$ contain an abnormality.
- Only in $M_3$ nothing is abnormal.
- Focus on models representing most normal situations.
- Accept a formula if it’s true in those models: here $flies$. 
Circumscription, ctd.

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- Given two interpretations over the same domain, \( I_1 \) and \( I_2 \). Let

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I_1 \leq I_2 \iff I_1[Ab] \subseteq I_2[Ab] \text{ for every } Ab \text{ predicate},
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- Define a new version of entailment:

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KB \models \leq \alpha \iff \text{ for every } I, I \models \alpha \text{ whenever } I \models KB \text{ and for no } I' < I \text{ we have } I' \models KB.
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- So \( \alpha \) must be true in all interpretations satisfying KB that are minimal in abnormalities.
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Models:

\[ M_1 = \{ \text{bird, ab, flies} \}, \ M_2 = \{ \text{bird, ab, } \neg \text{flies} \}, \ M_3 \text{ no longer a model.} \]

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Circumscription: 2nd order characterization

- Circumscription can be represented as a second order formula.

\[ T(P) \] first order formula containing predicate symbol \( P \). \( T(p) \) obtained from \( T(P) \) by replacing each occurrence of \( P \) by variable \( p \).

Abbreviations:

\[
\begin{align*}
P \leq Q & \text{ for } \forall x. P(x) \rightarrow Q(x) \\
P < Q & \text{ for } P \leq Q \text{ and not } Q \leq P
\end{align*}
\]

\( \text{Circ}(P, T(P)) \), the circumscription of \( P \) in \( T(P) \):

\[
T(P) \land \neg \exists p. (T(p) \land p < P)
\]

- Intuition: \( T(P) \) and there is no predicate smaller than \( P \) satisfying everything \( T \) says about \( P \).
- Theorem: \( T(Ab) \models \preceq q \) iff \( q \) consequence of \( \text{Circ}(Ab, T(Ab)) \).
Remarks

- Circumscription a skeptical approach: conflicting defaults cancel each other.
- Problem: 2nd order logic not even semi-decidable.
- Various results about when 2nd order formula has equivalent 1st order representation (Lifschitz).
- For restricted cases standard theorem provers can be used.
- Various more flexible variants of circumscription were defined: fixed predicates, preferences, ....
- They all have corresponding 2nd order formula.
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To represent defaults, Reiter uses rules of the form

\[ A : B_1, \ldots, B_n / C \]

where \( A, B_i, C \) are formulas.

Intuition: if \( A \) believed and each \( B_i \) consistent with beliefs, then infer \( C \).

Default theory: \((D, W)\), \( D \) set of defaults, \( W \) set of formulas representing what is known to be true.

Default theories generate extensions: acceptable sets of beliefs.

Main problem: cannot apply defaults constructively; consistency condition must hold with respect to final outcome.

Reiter’s fixpoint solution: guess the final outcome and verify that the guess was good.
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Motivation of fixpoint construction

- Properties an extension $E$ should satisfy
  1. should contain $W$ and be deductively closed,
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  3. no formula in $E$ without reasonable derivation from $W$, possibly using applicable defaults.

- (3) not achieved by considering minimal sets satisfying (1),(2).

Example

$D = \{ \text{prof}(x) : \text{teaches}(x)/\text{teaches}(x) \}$

$W = \{ \text{prof}(\text{gerd}) \}$

$Th(\{ \text{prof}(\text{gerd}), \neg \text{teaches(gerd)} \})$ minimal set satisfying (1),(2).

Obviously not intended: $\neg \text{teaches(gerd)}$ out of the blue.
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- Standard inference: iterative construction of closure; at each step apply inference rule applicable wrt. what was derived so far.
- What is inferred once remains conclusion forever.
- Not so for defaults: consistency at some stage may be lost later.

Example

\[ D = \{ p : q/r, \ p : s/s, \ s : \neg q/\neg q \} \]
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Sequence of sets generated by applicable defaults and deduction:
\[ E_0 = \{ p \}; \ E_1 = Th(\{ p, r, s \}); \ E_2 = Th(\{ p, r, s, \neg q \}) \]

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- Guess outcome of inference process; verify it’s justified.
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- Fixpoints of the operator then are what we are looking for.

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Let $\Delta = (D, W)$ be a default theory, $S$ a set of formulas. $\Gamma_\Delta(S)$ is the smallest set of formulas satisfying

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</tr>
</thead>
<tbody>
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• Types of defaults:
  
  • Normal: $p : q/q$. Normal default theories always have extensions.
  
  • Supernormal: $true : q/q$. Can model Poole systems.
  
  • Seminormal: $true : p \land q/q$. Used to encode preferences. Extensions may not exist.

• Extensions subset minimal: $E_1, E_2$ extensions $\Rightarrow E_1 \not\subseteq E_2$.

• $W$ inconsistent iff set of all formulas single extension.

• Defaults with open variables: usually viewed as schemata.
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- Answer sets (alias stable models for programs considered here) provide semantics for logic programs with \texttt{not}.
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- Default negation \texttt{not} interpreted procedurally: negation as failure.
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\begin{itemize}
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  \item \(a\) provable iff proof for \(b\) fails iff proof of \(a\) succeeds iff ...
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A (ground) normal logic program \( P \) is a collection of rules of the form

\[
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where \( A, B_i, C_j \) are ground atoms. \text{not } C \text{ reads: } C \text{ is not believed.}

- Answer set: atoms representing reasonable beliefs based on \( P \).
- Intuition similar to default logic:
  1. Each applicable rule applied.
  2. No atom without valid derivation.
- Simplifications: no set \( W \); beliefs fully determined by atoms.
- Identify rule with default \( B_1 \land \ldots \land B_n : \neg C_1, \ldots \neg C_m / A \) and strip unneeded parts off Reiter’s definition \( \Rightarrow \) GL-reduct.
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- $Cl(R)$ denotes the closure of a set of classical inference rules.
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Answer set programming

- Represent problem such that solutions are (parts of) answer sets.
- Commonly used method: generate and test:
  1. Generate candidate sets of atoms.
  2. Eliminate those not satisfying intended properties.
  3. Elimination via rules without head.
- Observation: if $P$ does not contain $q$, then
  
  $$ q \leftarrow \text{not } q, \text{body} $$

  eliminates answer sets satisfying body.
- Abbreviation: $\leftarrow \text{body}$. 
Variables in programs

- Definition of answer sets for propositional programs.
- Variables useful for problem descriptions.
- Rule with variables shorthand for all ground instances of the rule.
- ASP system: grounder + solver.
- Grounder produces ground instantiation of program, solver computes its answer sets.
Graph coloring

Example

Description of graph:
\(\text{node}(v_1), \ldots, \text{node}(v_n), \text{edge}(v_i, v_j), \ldots\)

Generate:
\[
\begin{align*}
\text{col}(X, r) & \leftarrow \text{node}(X), \neg \text{col}(X, b), \neg \text{col}(X, g) \\
\text{col}(X, b) & \leftarrow \text{node}(X), \neg \text{col}(X, r), \neg \text{col}(X, g) \\
\text{col}(X, g) & \leftarrow \text{node}(X), \neg \text{col}(X, r), \neg \text{col}(X, b)
\end{align*}
\]

Test:
\[
\leftarrow \text{edge}(X, Y), \text{col}(X, Z), \text{col}(Y, Z)
\]

Answer sets contain solution to problem!
Meeting scheduling

Example

Problem instance:

\[ \text{meeting}(m_1), \ldots, \text{meeting}(m_n) \]
\[ \text{time}(t_1), \ldots, \text{time}(t_s) \]
\[ \text{room}(r_1), \ldots, \text{room}(r_m) \]
\[ \text{person}(p_1), \ldots, \text{person}(p_k) \]
\[ \text{par}(p_1, m_1), \ldots, \text{par}(p_2, m_3), \ldots \]

Instance independent part, generate:

\[ \text{at}(M, T) \leftarrow \text{meeting}(M), \text{time}(T), \text{not} \neg \text{at}(M, T) \]
\[ \neg \text{at}(M, T) \leftarrow \text{meeting}(M), \text{time}(T), \text{not} \text{at}(M, T) \]
\[ \text{in}(M, R) \leftarrow \text{meeting}(M), \text{room}(R), \text{not} \neg \text{in}(M, R) \]
\[ \neg \text{in}(M, R) \leftarrow \text{meeting}(M), \text{room}(R), \text{not} \text{in}(M, R) \]
Example, ctd.

Each meeting has assigned time and room:

\[
\begin{align*}
timeassigned(M) & \leftarrow at(M, T) \\
roomassigned(M) & \leftarrow in(M, R) \\
\text{not } timeassigned(M) & \leftarrow meeting(M) \\
\text{not } roomassigned(M) & \leftarrow meeting(M)
\end{align*}
\]

No meeting has more than 1 time and room:

\[
\begin{align*}
\text{not } timeassigned(M) & \leftarrow meeting(M), at(M, T), at(M, T'), T \neq T' \\
\text{not } roomassigned(M) & \leftarrow meeting(M), in(M, R), in(M, R'), R \neq R'
\end{align*}
\]

Meetings at same time need different rooms:

\[
\begin{align*}
in(M, X), in(M', X) & \leftarrow at(M, T), at(M', T), M \neq M'
\end{align*}
\]

Meetings with same person need different times:

\[
\begin{align*}
par(P, M), par(P, M') & \leftarrow M \neq M', at(M, T), at(M', T)
\end{align*}
\]
Summary

- Presented some of the major approaches to nonmon.
  - Started with motivation and simple forms.
  - Sketched preferred subtheories, circumscription, default logic.
  - Finally presented definition of answer sets.
  - Focused on the main underlying ideas.
  - Many more approaches (autoepistemic logic, KLM), in particular some with implicit treatment of specificity and explicit preferences.
  - Current focus: ASP solvers; argumentation.
  - Preferences a natural aspect to bring in quantities.
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Suggested overview articles/books


THANK YOU!

G. Brewka, S. Woltran (Leipzig)
Nonmonotonic Reasoning
WS 2013/14
Suggested overview articles/books


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