This is as submitted to KR. Several typos will be fixed in the long version.

A Proofs

Proposition 1. *Let P be a program. Then, $NF(P)$ is in normal form and is strongly equivalent to P.*

Proof. (Berthold et al., 2019) proved that the first three steps produce an equivalent program. (Pearce and Valverde, 2008) proved that the fourth step produces an equivalent program. □

Proposition 2. *Let P be a program. Then, the normal form $NF(P)$ can be computed in PTIME.*

Proof. The first three steps of the algorithm require linear time, each rule must be passed once. The fourth step requires comparisons between every pair of rules. Hence, the complexity is quadratic. □

Proposition 3. *Given a program P in normal form over Σ, $X \subseteq Y \subseteq \Sigma$, and an atom $q \in \Sigma$, with $q \notin X$, and $\mathsf{occ}(P,q) = \langle R, R_0, R_1, R_2, R_3, R_4 \rangle$. Then the following equivalencies hold:*

$$\langle X, Y \rangle \nvDash P \Leftrightarrow \exists r \in R \cup R_1 \cup R_4 : \langle X, Y \rangle \nvDash r \quad (1)$$
$$\langle Xq, Yq \rangle \nvDash P \Leftrightarrow \exists r \in R \cup R_0 \cup R_2 : \langle Xq, Yq \rangle \nvDash r \quad (2)$$
$$\langle X, Yq \rangle \nvDash P \Leftrightarrow \exists r \in R \cup R_2 \cup R_3 \cup R_4 : \langle X, Yq \rangle \nvDash r \quad (3)$$

Proof.

(1) holds, since $\forall r \in R_0 \cup R_2 \cup R_3 : \langle X, Y \rangle \vDash r$
(2) holds, since $\forall r \in R_1 \cup R_3 \cup R_4 : \langle Xq, Yq \rangle \vDash r$
(3) holds, since $\forall r \in R_0 \cup R_1 : \langle X, Yq \rangle \vDash r$

□

Proposition 4. *Given a program P in normal form over Σ, $Y \subseteq \Sigma$, and an atom $q \in \Sigma$, with $q \notin Y$, and $\mathsf{occ}(P,q) = \langle R, R_0, R_1, R_2, R_3, R_4 \rangle$. Then the following equivalencies hold:*

$$\langle Y, Y \rangle \vDash P \Rightarrow \exists D \in \mathcal{D}^q_{as}(R_1 \cup R_4) : \langle Y, Y \rangle \nvDash \leftarrow D \quad (1)$$
$$\langle Yq, Yq \rangle \vDash P \Rightarrow \exists D \in \mathcal{D}^q_{as}(R_0 \cup R_2) : \langle Yq, Yq \rangle \nvDash \leftarrow D \quad (2)$$
$$\langle Y, Yq \rangle \vDash P \Rightarrow \exists D \in \mathcal{D}^q_{as}(R_3 \cup R_4) : \langle Y, Yq \rangle \nvDash \leftarrow D \quad (3)$$

In the case that $R = \emptyset$ the first and second implication hold in both directions.

Proof.

(1) by Prop. 3, $\langle Y, Y \rangle$ can only be contradicted by R, R_1 and R_4, if it is not contradicted by R_1 nor R_4, then it is contradicted by a $\leftarrow D$ s.t. $D \in \mathcal{D}^q_{as}(R_1 \cup R_4)$.
(2) by Prop. 3, $\langle Yq, Yq \rangle$ can only be contradicted by R, R_0 and R_2, if it is not contradicted by R_0 nor R_2, then it is contradicted by a $\leftarrow D$ s.t. $D \in \mathcal{D}^q_{as}(R_0 \cup R_2)$.
(3) by Prop. 3, $\langle Y, Y \rangle$ can only be contradicted by R, R_2, R_3 and R_4, if it is not contradicted by R_3 nor R_4, then it is contradicted by a $\leftarrow D$ s.t. $D \in \mathcal{D}^q_{as}(R_3 \cup R_4)$.

□

Proposition 5. *Given a rules r over Σ, and $X \subseteq Y \subseteq \Sigma$, the following statement holds:*

$$Y \vDash r \Leftrightarrow \langle X, Y \rangle \vDash \mathit{not\,not}\,r$$

Proof.

$$\begin{aligned} & \langle X, Y \rangle \vDash \mathit{not\,not}\,r \\ \Rightarrow\ & Y \vDash \mathit{not\,not}\,r \\ \Leftrightarrow\ & Y \vDash r \end{aligned}$$

$$\begin{aligned} & Y \vDash r \\ \Leftrightarrow\ & Y \vDash \mathit{not\,not}\,r \\ \Leftrightarrow\ & \{\mathit{not\,not}\,r\}^Y = \emptyset \\ \Rightarrow\ & X \vDash \{\mathit{not\,not}\,r\}^Y \\ \Rightarrow\ & \langle X, Y \rangle \vDash \mathit{not\,not}\,r \end{aligned}$$

□

Proposition 6. *Let r_1, r_2 be rules over Σ, and $X \subseteq Y \subseteq \Sigma$,*

$$Y \vDash r_1 \times r_2 \Leftrightarrow Y \vDash r_1 \vee Y \vDash r_2$$
$$X \vDash \{r_1 \times r_2\}^Y \Leftrightarrow X \vDash \{r_1\}^Y \vee X \vDash \{r_2\}^Y$$

Proof.

$$\begin{aligned} & Y \vDash r_1 \vee Y \vDash r_2 \\ \Leftrightarrow\ & Y \vDash H(r_1) \cup H(r_2) \leftarrow B(r_1) \cup B(r_2) \\ \Leftrightarrow\ & Y \vDash r_1 \times r_2 \end{aligned}$$

Then,

$$B^-(r_1 \times r_2) \cap Y = \emptyset \Leftrightarrow \left(B^-(r_1) \cup B^-(r_2)\right) \cap Y = \emptyset$$
$$B^{--}(r_1 \times r_2) \subseteq Y \Leftrightarrow B^{--}(r_1) \cup B^{--}(r_2) \subseteq Y$$

If $\{r_1\}^Y \neq \emptyset$ and $\{r_2\}^Y \neq \emptyset$, then

$$\begin{aligned} & X \vDash \{r_1\}^Y \ \vee\ X \vDash \{r_2\}^Y \\ \Leftrightarrow\ & X \vDash H(r_1) \leftarrow B^+(r_1) \vee X \vDash H(r_2) \leftarrow B^+(r_2) \\ \Leftrightarrow\ & X \vDash H(r_1) \cup H(r_2) \leftarrow B^+(r_1) \cup B^+(r_2) \\ \Leftrightarrow\ & X \vDash \{r_1 \times r_2\}^Y \end{aligned}$$

□

Proposition 7. *Let P_1, P_2 be programs over Σ, and $X \subseteq Y \subseteq \Sigma$, then*

$$Y \vDash P_1 \times P_2 \Leftrightarrow Y \vDash P_1 \vee Y \vDash P_2$$
$$X \vDash (P_1 \times P_2)^Y \Leftrightarrow X \vDash P_1^Y \vee X \vDash P_2^Y$$

Proof.

$$\begin{aligned} & Y \models P_1 \vee Y \models P_2 \\ \Leftrightarrow\ & \forall r_1 \in P_1 : Y \models r_1 \ \vee\ \forall r_2 \in P_2 : Y \models r_2 \\ \Leftrightarrow\ & \forall r \in P_1 \times P_2 : Y \models r \\ \Leftrightarrow\ & Y \models P_1 \times P_2 \end{aligned}$$

$$\begin{aligned} & X \models P_1^Y \vee X \models P_2^Y \\ \Leftrightarrow\ & \forall r_1 \in P_1^Y : X \models r_1 \ \vee\ \forall r_2 \in P_2^Y : X \models r_2 \\ \Leftrightarrow\ & \forall r \in (P_1 \times P_2)^Y : X \models r \\ \Leftrightarrow\ & X \models (P_1 \times P_2)^Y \end{aligned}$$

□

Proposition 8. *Given a program P in normal form over Σ, $X \subset Y \subseteq \Sigma$, and an atom $q \in \Sigma$, with $q \notin X$, and $\mathsf{occ}(P, q) = \langle R, R_0, R_1, R_2, R_3, R_4 \rangle$. Then the following equivalencies hold:*

$$\langle Y, Y \rangle \not\models P \Leftrightarrow \exists r \in R \cup R_1 \cup R_4 : \langle Y, Y \rangle \not\models r^{\setminus q} \quad (1)$$

$$\langle X, Y \rangle \not\models P \Leftrightarrow \exists r \in R \cup R_1 \cup R_4 : \langle X, Y \rangle \not\models r^{\setminus q} \quad (2)$$

$$\langle Yq, Yq \rangle \not\models P \Leftrightarrow \exists r \in R \cup R_0 \cup R_2 : \langle Y, Y \rangle \not\models r^{\setminus q} \quad (3)$$

$$\langle Y, Yq \rangle \not\models P \Leftrightarrow \langle Yq, Yq \rangle \not\models P \vee \exists r \in R_3 \cup R_4 : \langle Y, Y \rangle \not\models not\,not\,r^{\setminus q} \quad (4)$$

$$\langle Y, Yq \rangle \models P \Leftrightarrow \langle Yq, Yq \rangle \models P \wedge \exists D \in \mathcal{D}_{as}^q(R_3 \cup R_4) : \langle Y, Y \rangle \not\models \leftarrow D \quad (5)$$

$$\langle Xq, Yq \rangle \not\models P \Leftrightarrow \exists r \in R \cup R_0 \cup R_2 : \langle X, Y \rangle \not\models r^{\setminus q} \quad (6)$$

$$\langle X, Yq \rangle \not\models P \Leftrightarrow \langle Yq, Yq \rangle \not\models P \vee \exists r \in R \cup R_2 \cup R_3 \cup R_4 : \langle X, Y \rangle \not\models r^{\setminus q} \quad (7)$$

If additionally $R = \varnothing$, then

$$\langle Y, Y \rangle \models P \Leftrightarrow \exists D \in \mathcal{D}_{as}^q(R_1 \cup R_4) : \langle Y, Y \rangle \not\models \leftarrow D \quad (8)$$

$$\langle Yq, Yq \rangle \models P \Leftrightarrow \exists D \in \mathcal{D}_{as}^q(R_0 \cup R_2) : \langle Y, Y \rangle \not\models \leftarrow D \quad (9)$$

For all $r_0 \in R_0$:

$$\langle Yq, Yq \rangle \not\models r_0 \Leftrightarrow \langle Y, Y \rangle \not\models not\,not\,r_0^{\setminus q}. \quad (10)$$

For all $r_2 \in R_2$:

$$\langle Yq, Yq \rangle \not\models r_2 \Leftrightarrow \langle Y, Yq \rangle \not\models r_2, \text{ and} \quad (11)$$

$$\langle Xq, Yq \rangle \not\models r_2 \Leftrightarrow \langle X, Yq \rangle \not\models r_2. \quad (12)$$

For all $r_4 \in R_4$, if $Y \models r_4$:

$$\langle Y, Yq \rangle \not\models r_4 \Leftrightarrow \langle Y, Y \rangle \not\models r_4, \text{ and} \quad (13)$$

$$\langle X, Yq \rangle \not\models r_4 \Leftrightarrow \langle X, Y \rangle \not\models r_4. \quad (14)$$

For all $r \in R$:

$$\langle Yq, Yq \rangle \models r \Leftrightarrow \langle Y, Yq \rangle \models r \Leftrightarrow \langle Y, Y \rangle \models r, \text{ and} \quad (15)$$

$$\langle Xq, Yq \rangle \models r \Leftrightarrow \langle X, Yq \rangle \models r \Leftrightarrow \langle X, Y \rangle \models r. \quad (16)$$

Proof.

(1) holds, since $\forall r \in R_0 \cup R_2 \cup R_3 : \langle Y, Y \rangle \models r$ and $\forall r \in R \cup R_1 \cup R_4 : \langle Y, Y \rangle \models r \Leftrightarrow \langle Y, Y \rangle \models r^{\setminus q}$

(2) holds, since $\forall r \in R_0 \cup R_2 \cup R_3 : \langle X, Y \rangle \models r$ and $\forall r \in R \cup R_1 \cup R_4 : \langle X, Y \rangle \models r \Leftrightarrow \langle X, Y \rangle \models r^{\setminus q}$

(3) holds, since $\forall r \in R_1 \cup R_4 : \langle Yq, Yq \rangle \models r$ and $\forall r \in R \cup R_0 \cup R_2 : \langle Yq, Yq \rangle \not\models r \Leftrightarrow \langle Y, Y \rangle \not\models r^{\setminus q}$

(4) $\langle Yq, Yq \rangle \not\models P \Rightarrow \langle Y, Yq \rangle \not\models P$, now assuming that $\langle Yq, Yq \rangle \models P$ we prove:

$$\langle Y, Yq \rangle \not\models P \Leftrightarrow \exists r \in R_3 \cup R_4 : \langle Y, Y \rangle \not\models not\,not\,r^{\setminus q}$$

$$\begin{aligned} & \langle Yq, Yq \rangle \models P \\ \Rightarrow\ & \forall r \in R \cup R_2 : \langle Yq, Yq \rangle \models r \\ \overset{(11),(15)}{\Rightarrow}\ & \forall r \in R \cup R_2 : \langle Y, Yq \rangle \models r \end{aligned} \quad \text{(i)}$$

$$\text{also, trivially: } \forall r \in R_0 \cup R_1 : \langle Y, Yq \rangle \models r \quad \text{(ii)}$$

$$\begin{aligned} & \langle Y, Y \rangle \not\models P \\ \overset{\text{(i), (ii)}}{\Leftrightarrow}\ & \exists r \in R_3 \cup R_4 : \langle Y, Y \rangle \not\models r \\ \overset{\text{Prop. 5}}{\Leftrightarrow}\ & \exists r \in R_3 \cup R_4 : \langle Y, Y \rangle \not\models not\,not\,r \\ \overset{q\text{ has no effect}}{\Leftrightarrow}\ & \exists r \in R_3 \cup R_4 : \langle Y, Y \rangle \not\models not\,not\,r^{\setminus q} \end{aligned}$$

(5) This statement is the opposite of (4). Assuming that $\langle Yq, Yq \rangle \models P$:
$\langle Y, Yq \rangle$ can only be contradicted by rules in $R_3 \cup R_4$. Then $\langle Y, Yq \rangle \models R_3 \cup R_4 \Leftrightarrow \exists D \in \mathcal{D}_{as}^q(R_3 \cup R_4) : \langle Y, Yq \rangle \not\models \leftarrow D$. D does not contain q, therefore, $\langle Y, Y \rangle \not\models \leftarrow D$.

(6) holds, since $\forall r \in R_1 \cup R_3 \cup R_4 : \langle Xq, Yq \rangle \models r$ and $\forall r \in R \cup R_0 \cup R_2 : \langle Xq, Yq \rangle \models r \Leftrightarrow \langle X, Y \rangle \models r^{\setminus q}$

(7) $\langle Yq, Yq \rangle \not\models P \Rightarrow \langle Xq, Yq \rangle \not\models P$, now assuming that $\langle Yq, Yq \rangle \models P$ we prove:

$$\langle Xq, Yq \rangle \not\models P \Leftrightarrow \exists r \in R \cup R_0 \cup R_2 : \langle X, Y \rangle \not\models r^{\setminus q}$$

$$\begin{aligned} & \langle Yq, Yq \rangle \models P \\ \Rightarrow\ & \forall r \in R_0 : \langle Yq, Yq \rangle \models r \\ \Rightarrow\ & \forall r \in R_0 : \langle X, Yq \rangle \models r \end{aligned} \quad \text{(i)}$$

$$\text{also, trivially: } \forall r \in R_1 : \langle X, Yq \rangle \models r \quad \text{(ii)}$$

$$\begin{aligned} & \langle X, Yq \rangle \not\models P \\ \overset{\text{(i), (ii)}}{\Leftrightarrow}\ & \exists r \in R \cup R_2 \cup R_3 \cup R_4 : \langle X, Yq \rangle \not\models r \\ \Leftrightarrow\ & \exists r \in R \cup R_2 \cup R_3 \cup R_4 : \langle X, Y \rangle \not\models r^{\setminus q} \end{aligned}$$

(8) $\langle Y, Y \rangle$ can only be contradicted by rules in $R \cup R_1 \cup R_4$, since $R = \varnothing$, $\langle Y, Y \rangle \models P \Leftrightarrow \langle Y, Y \rangle \models R_1 \cup R_4 \Leftrightarrow \exists D \in \mathcal{D}_{as}^q(R_1 \cup R_4) : Y, Y \not\models \leftarrow D$

(9) $\langle Yq, Yq \rangle$ can only be contradicted by rules in $R \cup R_0 \cup R_2$, since $R = \varnothing$, $\langle Yq, Yq \rangle \models P \Leftrightarrow \langle Yq, Yq \rangle \models R_0 \cup R_2 \Leftrightarrow \exists D \in \mathcal{D}_{as}^q(R_0 \cup R_2) : Yq, Yq \not\models \leftarrow D$. D does not contain q, therefore, $\exists D \in \mathcal{D}_{as}^q(R_0 \cup R_2) : \langle Y, Y \rangle \not\models \leftarrow D$

(10) holds by Prop. 5 and since $\langle Yq, Yq\rangle \models not\,not\,r_0 \Leftrightarrow \langle Y, Y\rangle \models not\,not\,r_0^{\setminus q}$.

(11) $Y \models \{r_2\}^{Y\cup\{q\}} \Leftrightarrow Y \cup \{q\} \models \{r_2\}^{Y\cup\{q\}}$, since $\{r_2\}^{Y\cup\{q\}}$ contains no appearance of q.

(12) $X \models \{r_2\}^{Y\cup\{q\}} \Leftrightarrow X \cup \{q\} \models \{r_2\}^{Y\cup\{q\}}$, since $\{r_2\}^{Y\cup\{q\}}$ contains no appearance of q.

(13) $Y \cup \{q\} \models r_4$ and $Y \models \{r_4\}^Y \Leftrightarrow Y \models \{r_4\}^{Y\cup\{q\}}$

(14) $Y \cup \{q\} \models r_4$ and $X \models \{r_4\}^Y \Leftrightarrow X \models \{r_4\}^{Y\cup\{q\}}$

(15) holds, since r contains no appearance of q.

(16) holds, since r contains no appearance of q.

□

Proposition 9. *Given a program P over Σ, an atom $q \in \Sigma$, and sets X and Y, s.t. $X \subset Y \subseteq \Sigma\setminus\{q\}$, then*

$$\langle Y, Y\rangle \models \mathsf{f}_R^+(P, q) \Leftrightarrow \{q\} \in Rel^Y_{\langle P,\{q\}\rangle}$$

If $\{q\} \in Rel^Y_{\langle P,\{q\}\rangle}$, then

$$\langle X, Y\rangle \models \mathsf{f}_R^+(P, q) \Leftrightarrow \langle Xq, Yq\rangle \models P \vee \langle X, Yq\rangle \models P$$

Proof.

$$\begin{aligned}
& \{q\} \notin Rel^Y_{\langle P,\{q\}\rangle} \\
\Leftrightarrow\ & \langle Yq, Yq\rangle \notin \mathcal{HT}(P) \\
\vee\ & \langle Y, Yq\rangle \in \mathcal{HT}(P) \\
\overset{\text{Prop. 8}}{\Leftrightarrow}\ & \exists r \in R \cup R_0 \cup R_2 \text{ s.t. } \langle Y, Y\rangle \not\models r^{\setminus q} \\
\vee\ & \exists D \in \mathcal{D}^q_{as}(R_3 \cup R_4) \text{ s.t. } \langle Y, Y\rangle \not\models \leftarrow D \\
\Leftrightarrow\ & \exists r \in R \cup R_2 \text{ s.t. } \langle Y, Y\rangle \not\models r^{\setminus q} \\
\vee\ & \exists r_0 \in R_0 \text{ s.t. } \langle Y, Y\rangle \not\models \leftarrow not\,not\,r_0^{\setminus q} \\
\vee\ & \exists D \in \mathcal{D}^q_{as}(R_3 \cup R_4) \text{ s.t. } \langle Y, Y\rangle \not\models \leftarrow D \\
\overset{\text{Def. 10}}{\Leftrightarrow}\ & \exists r \in \mathsf{f}_R^+(P, q) \text{ s.t. } \langle Y, Y\rangle \not\models r \\
\Leftrightarrow\ & \langle Y, Y\rangle \notin \mathcal{HT}(\mathsf{f}_R^+(P, q))
\end{aligned}$$

$$\begin{aligned}
& \langle Xq, Yq\rangle \notin \mathcal{HT}(P) \\
& \wedge \langle X, Yq\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Prop. 8}}{\Leftrightarrow}\ & \exists r \in R \cup R_2 \text{ s.t. } \langle X, Y\rangle \not\models r^{\setminus q} \\
& \vee \big(\exists r_0 \in R_0 \text{ s.t. } \langle X, Y\rangle \not\models r_0^{\setminus q} \wedge \\
& \quad \exists r' \in R_3 \cup R_4 \text{ s.t. } \langle X, Y\rangle \not\models r'^{\setminus q}\big) \\
\overset{\text{Prop. 6}}{\Leftrightarrow}\ & \exists r \in R \cup R_2 \text{ s.t. } \langle X, Y\rangle \not\models r^{\setminus q} \\
& \vee\ \exists r_0 \in R_0 \exists r' \in \cup R_3 \cup R_4 \text{ s.t.} \\
& \quad \langle X, Y\rangle \not\models (r_0 \times r')^{\setminus q} \\
\overset{\text{Def. 10}}{\Leftrightarrow}\ & \exists r \in \mathsf{f}_R^+(P, q) \text{ s.t. } \langle X, Y\rangle \not\models r \\
\Leftrightarrow\ & \langle X, Y\rangle \notin \mathcal{HT}(\mathsf{f}_R^+(P, q))
\end{aligned}$$

□

Proposition 10. *Given a program P over Σ, an atom $q \in \Sigma$, and sets X and Y, s.t. $X \subset Y \subseteq \Sigma\setminus\{q\}$, then*

$$\langle Y, Y\rangle \models \mathsf{f}_R^-(P, q) \Leftrightarrow \varnothing \in Rel^Y_{\langle P,\{q\}\rangle}$$

If $\varnothing \in Rel^Y_{\langle P,\{q\}\rangle}$, then

$$\langle X, Y\rangle \models \mathsf{f}_R^-(P, q) \Leftrightarrow \langle X, Y\rangle \models P$$

Proof.

$$\begin{aligned}
& \langle X, Y\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Prop. 6}}{\Leftrightarrow}\ & \exists r \in R \cup R_1 \cup R_4 \cup R \text{ s.t. } \langle X, Y\rangle \not\models r^{\setminus q} \\
\overset{\text{Def. 11}}{\Leftrightarrow}\ & \exists r \in \mathsf{f}_R^-(P, q) \text{ s.t. } \langle X, Y\rangle \not\models r \\
\Leftrightarrow\ & \langle X, Y\rangle \notin \mathcal{HT}(\mathsf{f}_R^-(P, q))
\end{aligned}$$

$$\begin{aligned}
& \varnothing \notin Rel^Y_{\langle P,\{q\}\rangle} \\
\Leftrightarrow\ & \langle Y, Y\rangle \notin \mathcal{HT}(P) \\
\overset{\text{like above}}{\Leftrightarrow}\ & \langle Y, Y\rangle \notin \mathcal{HT}(\mathsf{f}_R^-(P, q))
\end{aligned}$$

□

Proposition 11. *Given a program P over Σ, and sets X, Y, A and V, s.t. $A \subseteq V \subseteq \Sigma$, and $X \subseteq Y \subseteq \Sigma\setminus V$, then*

$$\langle Y, Y\rangle \models \mathsf{f}_R^A(P, V) \Leftrightarrow A \in Rel^Y_{\langle P,V\rangle}$$

If $A \in Rel^Y_{\langle P,V\rangle}$, then

$$\langle X, Y\rangle \models \mathsf{f}_R^A(P, V) \Leftrightarrow \exists A'' \subseteq A : \langle XA'', YA\rangle \models P$$

Proof. Proof by induction.
Base. Let $V = \varnothing$, then, $A = \varnothing$, and therefore

$$\begin{aligned}
& A \in Rel^Y_{\langle P,V\rangle} \\
\Leftrightarrow\ & \langle YA, YA\rangle \in \mathcal{HT}(P) \\
& \wedge \forall A' \subset A : \langle YA', YA\rangle \notin \mathcal{HT}(P) \\
\Leftrightarrow\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}_R^A(P, V)) = \mathcal{HT}(P)
\end{aligned}$$

Hypothesis. We assume that the statement holds for V containing n atoms.
Step. Let $V = \{q_1, \ldots, q_{n+1}\}$ (Recall, that $A^{\setminus q_1} = A\setminus\{q_1\}$.)

- If $q_1 \in A$, then

$$
\begin{aligned}
& A \in Rel^Y_{\langle P,V\rangle} \\
\overset{\text{Def.}}{\Leftrightarrow}\ & \langle YA, YA\rangle \in \mathcal{HT}(P) \\
& \wedge \forall A' \subset A : \langle YA', YA\rangle \notin \mathcal{HT}(P) \\
\Leftrightarrow\ & \langle YA^{\setminus q_1} q_1, YA^{\setminus q_1} q_1\rangle \in \mathcal{HT}(P) \\
& \wedge \langle YA^{\setminus q_1}, YA^{\setminus q_1} q_1\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA' q_1, YA^{\setminus q_1} q_1\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA', YA^{\setminus q_1} q_1\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Prop. 9}}{\Leftrightarrow}\ & \langle YA^{\setminus q_1}, YA^{\setminus q_1}\rangle \in \mathcal{HT}(\mathsf{f}^+_R(P, q_1)) \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA', YA^{\setminus q_1}\rangle \notin \mathcal{HT}(\mathsf{f}^+_R(P, q_1)) \\
\overset{P' := \mathsf{f}^+_R(P,q_1)}{\Leftrightarrow}\ & \langle YA^{\setminus q_1}, YA^{\setminus q_1}\rangle \in \mathcal{HT}(P') \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA', YA^{\setminus q_1}\rangle \notin \mathcal{HT}(P') \\
\overset{\text{Def.}}{\Leftrightarrow}\ & A^{\setminus q_1} \in Rel^Y_{\langle P', V\setminus\{q_1\}\rangle} \\
\overset{\text{Hypothesis}}{\Leftrightarrow}\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}^{A^{\setminus q_1}}_R(P', V\setminus\{q_1\})) \\
\overset{P' = \mathsf{f}^+_R(P,q_1)}{\Leftrightarrow}\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}^{A^{\setminus q_1}}_R(\mathsf{f}^+_R(P, q_1), V\setminus\{q_1\}) \\
\overset{\text{Def. 12}}{\Leftrightarrow}\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}^A_R(P, V))
\end{aligned}
$$

- If $q_1 \notin A$, then

$$
\begin{aligned}
& A \in Rel^Y_{\langle P,V\rangle} \\
\overset{\text{Def.}}{\Leftrightarrow}\ & \langle YA, YA\rangle \in \mathcal{HT}(P) \\
& \wedge \forall A' \subset A : \langle YA', YA\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Prop. 10}}{\Leftrightarrow}\ & \langle YA, YA\rangle \in \mathcal{HT}(\mathsf{f}^-_R(P, q_1)) \\
& \wedge \forall A' \subset A : \langle YA', YA\rangle \notin \mathcal{HT}(\mathsf{f}^-_R(P, q_1)) \\
\overset{P' := \mathsf{f}^-_R(P,q_1)}{\Leftrightarrow}\ & \langle YA, YA\rangle \in \mathcal{HT}(P') \\
& \wedge \forall A' \subset A : \langle YA', YA\rangle \notin \mathcal{HT}(P') \\
\overset{\text{Def.}}{\Leftrightarrow}\ & A \in Rel^Y_{\langle P', V\setminus\{q_1\}\rangle} \\
\overset{\text{Hypothesis}}{\Leftrightarrow}\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}^A_R(P', V\setminus\{q_1\}) \\
\overset{P' = \mathsf{f}^-_R(P,q_1)}{\Leftrightarrow}\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}^A_R(\mathsf{f}^-_R(P, q_1), V\setminus\{q_1\}) \\
\overset{\text{Def. 12}}{\Leftrightarrow}\ & \langle Y, Y\rangle \in \mathcal{HT}(\mathsf{f}^A_R(P, V))
\end{aligned}
$$

The second statement can be proven analogously. □

Theorem 1.

$$\mathsf{f}_R \in \mathsf{F}_\mathsf{R}$$

Proof. We have to show that for all P and $V \subseteq \Sigma$

$$\mathcal{HT}(\mathsf{f}_R(P, V)) = \{\langle X, Y\rangle \mid Y \subseteq \Sigma(P)\setminus V \wedge X \in \bigcup \mathcal{R}^Y_{\langle P,V\rangle}\}.$$

Given $Y \subseteq \Sigma(P)\setminus V$

$$
\begin{aligned}
& \langle X, Y\rangle \in \mathcal{HT}(\mathsf{f}_R(P, V)) \\
\overset{\text{Def. 13}}{\Leftrightarrow}\ & \langle X, Y\rangle \in \mathcal{HT}(NF(\bigtimes_{A\subseteq V} \mathsf{f}^A_R(P, V))) \\
\Leftrightarrow\ & \langle X, Y\rangle \in \mathcal{HT}(\bigtimes_{A\subseteq V} \mathsf{f}^A_R(P, V)) \\
\overset{\text{Prop. 7}}{\Leftrightarrow}\ & \exists A \subseteq V : \langle X, Y\rangle \in \mathcal{HT}(\mathsf{f}^A_R(P, V)) \\
\overset{\text{Prop. 11}}{\Leftrightarrow}\ & \exists A \in Rel^Y_{\langle X,Y\rangle} \text{ s.t.} \\
& \exists A'' \subseteq A : \langle XA'', YA\rangle \in \mathcal{HT}(P)
\end{aligned}
$$

□

Proposition 12. *Given a program P over Σ, an atom $q \in \Sigma$, and sets X and Y, s.t. $X \subseteq Y \subseteq \Sigma\setminus\{q\}$, then*

$$
\begin{aligned}
& \{q\} \in Rel^Y_{\langle P,\{q\}\rangle} \wedge \langle Xq, Yq\rangle \not\models P \wedge \langle X, Yq\rangle \not\models P \\
& \Leftrightarrow \langle X, Y\rangle \not\models \mathsf{f}^+_W(P, q)
\end{aligned}
$$

Proof.

$$
\begin{aligned}
& \langle Yq, Yq\rangle \in \mathcal{HT}(P) \\
\Leftrightarrow\ & \exists D \in \mathcal{D}^q_{as}(R_0 \cup R_2) \text{ s.t. } \langle Y, Y\rangle \not\models \leftarrow D \\
\Leftrightarrow\ & \exists D \in \mathcal{D}^q_{as}(R_0 \cup R_2) \text{ s.t. } \langle X, Y\rangle \not\models \leftarrow D
\end{aligned}
$$

$$
\begin{aligned}
& \langle Yq, Yq\rangle \in \mathcal{HT}(P) \wedge \langle Y, Yq\rangle \notin \mathcal{HT}(P) \\
\Leftrightarrow\ & \exists r \in R_3 \cup R_4 \text{ s.t. } \langle Y, Y\rangle \not\models r \\
\Leftrightarrow\ & \exists r \in R_3 \cup R_4 \text{ s.t. } \langle X, Y\rangle \not\models\ not\, not\, r^{\setminus q}
\end{aligned}
$$

$$
\begin{aligned}
& \langle Xq, Yq\rangle \notin \mathcal{HT}(P) \wedge \langle X, Yq\rangle \notin \mathcal{HT}(P) \\
\Leftrightarrow\ & \alpha \ \vee\ \beta \\
& \text{where}
\end{aligned}
$$

$$
\begin{aligned}
\alpha &= \exists r_0 \in R_0 \text{ s.t. } \langle Xq, Yq\rangle \not\models r_0 \\
&\quad \wedge \exists r' \in R_3 \cup R_4 \text{ s.t. } \langle X, Yq\rangle \not\models r' \\
\beta &= \exists r \in R \cup R_2 \text{ s.t. } \langle X, Y\rangle \not\models r
\end{aligned}
$$

Then equivalently

$$\langle X, Y\rangle \not\models r_0^{\setminus q} \times r'^{\setminus q} \times not\, not\, r^{\setminus q} \times \leftarrow D, \text{ or} \qquad (\alpha)$$

$$\langle X, Y\rangle \not\models r^{\setminus q} \times not\, not\, r'^{\setminus q} \times \leftarrow D \qquad (\beta)$$

which are derived by f^+_W. □

Proposition 13. *Given a program P over Σ, an atom $q \in \Sigma$, and sets X and Y, s.t. $X \subseteq Y \subseteq \Sigma\setminus\{q\}$, then*

$$
\begin{aligned}
& \varnothing \in Rel^Y_{\langle P,\{q\}\rangle} \wedge \langle X, Y\rangle \not\models P \\
& \Leftrightarrow \langle X, Y\rangle \not\models \mathsf{f}^-_W(P, q)
\end{aligned}
$$

Proof.

$$
\begin{aligned}
& \varnothing \in Rel^{Y}_{\langle P,V\rangle} \\
\Leftrightarrow\; & \langle Y,Y\rangle \in \mathcal{HT}(P) \\
\Leftrightarrow\; & \exists D \in \mathcal{D}^{q}_{as}(R_1 \cup R_4) \text{ s.t. } \langle Y,Y\rangle \not\models \leftarrow D \\
\Leftrightarrow\; & \exists D \in \mathcal{D}^{q}_{as}(R_1 \cup R_4) \text{ s.t. } \langle X,Y\rangle \not\models \leftarrow D
\end{aligned}
$$

$$
\begin{aligned}
& \langle X,Y\rangle \notin \mathcal{HT}(P) \\
\Leftrightarrow\; & \exists r \in R \cup R_1 \cup R_4 \text{ s.t. } \langle X,Y\rangle \not\models r \\
\Leftrightarrow\; & \exists r \in R \cup R_1 \cup R_4 \text{ s.t. } \langle X,Y\rangle \not\models r^{\setminus q}
\end{aligned}
$$

Then equivalently

$$\langle X,Y\rangle \not\models r^{\setminus q} \times \leftarrow D$$

which is derived by f^{-}_{W}. □

Proposition 14. *Given a program P over Σ, and sets X, Y, A and V, s.t. $A \subseteq V \subseteq \Sigma$, $X \subseteq Y \subseteq \Sigma \setminus V$, and $R = \{r \in P \mid V \cap \Sigma(r) = \varnothing\} = \varnothing$, then*

$$
\begin{aligned}
& A \in Rel^{Y}_{\langle P,V\rangle} \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \not\models P \\
& \quad \Leftrightarrow \langle X,Y\rangle \not\models \mathsf{f}^{A}_{W}(P,V)
\end{aligned}
$$

Proof. Proof by induction.
Base.
Let $V = \{q_1\}$, then $\mathsf{f}^{A}_{W}(P,V) = \mathsf{f}^{+}_{W}(P,V)$, or $\mathsf{f}^{A}_{W}(P,V) = \mathsf{f}^{-}_{W}(P,V)$. Respectively, the statement follows from Prop. 12 or Prop. 13.
Hypothesis. We assume that the statement holds for V containing n atoms.
Step. Let $V = \{q_1,\dots,q_{n+1}\}$
If $q_1 \in A$, then

$$
\begin{aligned}
& A \in Rel^{Y}_{\langle P,V\rangle} \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Def.}}{\Leftrightarrow}\; & \langle YA,YA\rangle \in \mathcal{HT}(P) \\
& \wedge \forall A' \subset A : \langle YA',YA\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(P) \\
\Leftrightarrow\; & \langle YA^{\setminus q_1}q_1,YA^{\setminus q_1}q_1\rangle \in \mathcal{HT}(P) \\
& \wedge \langle YA^{\setminus q_1},YA^{\setminus q_1}q_1\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA'q_1,YA^{\setminus q_1}q_1\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA',YA^{\setminus q_1}q_1\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A'' \subseteq A^{\setminus q_1} : \langle XA''q_1,YA^{\setminus q_1}q_1\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A'' \subseteq A^{\setminus q_1} : \langle XA'',YA^{\setminus q_1}q_1\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Prop. 12}}{\Leftrightarrow}\; & \langle YA^{\setminus q_1},YA^{\setminus q_1}\rangle \in \mathcal{HT}(\mathsf{f}^{+}_{W}(P,q_1)) \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA',YA^{\setminus q_1}\rangle \notin \mathcal{HT}(\mathsf{f}^{+}_{W}(P,q_1)) \\
& \wedge \forall A'' \subseteq A^{\setminus q_1} : \langle XA'',YA^{\setminus q_1}\rangle \notin \mathcal{HT}(\mathsf{f}^{+}_{W}(P,q_1)) \\
\overset{P' := \mathsf{f}^{+}_{W}(P,q_1)}{\Leftrightarrow}\; & \langle YA^{\setminus q_1},YA^{\setminus q_1}\rangle \in \mathcal{HT}(P') \\
& \wedge \forall A' \subset A^{\setminus q_1} : \langle YA',YA^{\setminus q_1}\rangle \notin \mathcal{HT}(P') \\
& \wedge \forall A'' \subseteq A^{\setminus q_1} : \langle XA'',YA^{\setminus q_1}\rangle \notin \mathcal{HT}(P') \\
\overset{\text{Def.}}{\Leftrightarrow}\; & A^{\setminus q_1} \in Rel^{Y}_{\langle P',V\setminus\{q_1\}\rangle} \\
& \wedge \forall A'' \subseteq A^{\setminus q_1} : \langle XA'',YA^{\setminus q_1}\rangle \notin \mathcal{HT}(P') \\
\overset{\text{Hypothesis}}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}^{A^{\setminus q_1}}_{R}(P',V\setminus\{q_1\})) \\
\overset{P' = \mathsf{f}^{+}_{W}(P,q_1)}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}^{A^{\setminus q_1}}_{R}(\mathsf{f}^{+}_{W}(P,q_1),V\setminus\{q_1\})) \\
\overset{\text{Def. 16}}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}^{A}_{W}(P,V))
\end{aligned}
$$

If $q_1 \notin A$, then

$$
\begin{aligned}
& A \in Rel^{Y}_{\langle P,V\rangle} \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Def.}}{\Leftrightarrow}\; & \langle YA,YA\rangle \in \mathcal{HT}(P) \\
& \wedge \forall A' \subset A : \langle YA',YA\rangle \notin \mathcal{HT}(P) \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(P) \\
\overset{\text{Prop. 13}}{\Leftrightarrow}\; & \langle YA,YA\rangle \in \mathcal{HT}(\mathsf{f}^{-}_{W}(P,q_1)) \\
& \wedge \forall A' \subset A : \langle YA',YA\rangle \notin \mathcal{HT}(\mathsf{f}^{-}_{W}(P,q_1)) \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(\mathsf{f}^{-}_{W}(P,q_1)) \\
\overset{P' := \mathsf{f}^{-}_{W}(P,q_1)}{\Leftrightarrow}\; & \langle YA,YA\rangle \in \mathcal{HT}(P') \\
& \wedge \forall A' \subset A : \langle YA',YA\rangle \notin \mathcal{HT}(P') \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(P') \\
\overset{\text{Def.}}{\Leftrightarrow}\; & A \in Rel^{Y}_{\langle P',V\setminus\{q_1\}\rangle} \\
& \wedge \forall A'' \subseteq A : \langle XA'',YA\rangle \notin \mathcal{HT}(P')
\end{aligned}
$$

$$
\begin{aligned}
\overset{\text{Hypothesis}}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}^{A}_{R}(P',V\setminus\{q_1\})) \\
\overset{P' = \mathsf{f}^{-}_{W}(P,q_1)}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}^{A}_{R}(\mathsf{f}^{-}_{W}(P,q_1),V\setminus\{q_1\})) \\
\overset{\text{Def. 16}}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}^{A}_{W}(P,V))
\end{aligned}
$$

□

Proposition 15. *Given a program P over Σ in normal form, three sets of atoms X, Y and V, s.t. $X \subseteq Y \subseteq \Sigma$, $V \subseteq \Sigma$, and $R = \{r \in P \mid V \cap \Sigma(r) = \varnothing\} = \varnothing$, then*

$$
\begin{aligned}
& \exists A \in Rel^{Y}_{\langle P,V\rangle} \text{ s.t. } \forall A'' \subseteq A. \langle XA'',YA\rangle \not\models P \\
& \quad \Leftrightarrow \langle X,Y\rangle \not\models \mathsf{f}_{W}(P,V)
\end{aligned}
$$

Proof.

$$
\begin{aligned}
& \langle X,Y\rangle \notin \mathcal{HT}(\mathsf{f}_{W}(P,V)) \\
\overset{\text{Def. 17}}{\Leftrightarrow}\; & \langle X,Y\rangle \notin \mathcal{HT}(\bigcup_{A \subseteq V} \mathsf{f}^{A}_{W}(P,V))
\end{aligned}
$$

$$\Leftrightarrow\ \exists A \subseteq V \text{ s.t. } \langle X, Y\rangle \notin \mathcal{HT}(\mathsf{f}_W^A(P,V))$$

$$\overset{\text{Prop. 14}}{\Leftrightarrow}\ \exists A \in Rel^Y_{\langle P,V\rangle} \text{ s.t. } \forall A'' \subseteq A. \langle XA'', YA\rangle \notin \mathcal{HT}(P)$$

□

Theorem 2.

$$\mathsf{f}^*_{SP} \in \mathsf{F_{SP}}$$

Proof. Given a program P over Σ and $X \subseteq Y \subseteq \Sigma$.

$$\begin{aligned}
&\langle X, Y\rangle \in \mathcal{HT}(\mathsf{f}^*_{SP}(P,V))\\
\overset{\text{Def. 18}}{\Leftrightarrow}\ &\langle X, Y\rangle \in \mathcal{HT}(\mathsf{f}_R(P,V) \cup \mathsf{f}_W(P,V))\\
\Leftrightarrow\ &\langle X, Y\rangle \in \mathcal{HT}(\mathsf{f}_R(P,V))\\
&\wedge \langle X, Y\rangle \in \mathcal{HT}(\mathsf{f}_W(P,V))\\
\overset{\text{Thm. 1, Prop. 15}}{\Leftrightarrow}\ &\exists A \in Rel^Y_{\langle X,Y\rangle} \text{ s.t.}\\
&\exists A'' \subseteq A : \langle XA'', YA\rangle \notin \mathcal{HT}(P)\\
&\wedge \forall A \in Rel^Y_{\langle P,V\rangle} \text{ s.t.}\\
&\exists A'' \subseteq A. \langle XA'', YA\rangle \notin \mathcal{HT}(P)\\
\Leftrightarrow\ &\exists A \in Rel^Y_{\langle X,Y\rangle}\\
&\wedge \forall A \in Rel^Y_{\langle P,V\rangle} \text{ s.t.}\\
&\exists A'' \subseteq A. \langle XA'', YA\rangle \notin \mathcal{HT}(P)
\end{aligned}$$

with other words, for all programs P and $V \subseteq \Sigma$,

$$\mathcal{HT}(\mathsf{f}^*_{SP}(P,V)) = \{\langle X,Y\rangle \mid Y \subseteq \Sigma(P)\backslash V \wedge X \in \bigcap \mathcal{R}^Y_{\langle P,V\rangle}\}.$$

Hence $\mathsf{f}^*_{SP} \in \mathsf{F_{SP}}$ holds. □

Proposition 16. *Let P_1, P_2 and R be a programs over Σ,*

$$(P_1 \cup R) \times (P_2 \cup R) \equiv (P_1 \times P_2) \cup R.$$

Proof.

$$\mathcal{HT}((P_1 \cup R) \times (P_2 \cup R)) \subseteq \mathcal{HT}(((P_1 \times P_2) \cup R))$$

is easily witnessed by the fact that

$$(P_1 \cup R) \times (P_2 \cup R) \supseteq (P_1 \times P_2) \cup R$$

$$\mathcal{HT}((P_1 \cup R) \times (P_2 \cup R)) \supseteq \mathcal{HT}(((P_1 \times P_2) \cup R))$$

on the other hand holds, since $(P_1 \cup R) \times (P_2 \cup R)$ contains all R and any rule that is created by conjoining an $r \in R$ with an $r_1 \in P_1$ or $r_2 \in P_2$ is subsumed by r. □

We recall the distance measure between two rules (resp. programs) from (Berthold et al., 2019).

Definition 20. *Let r and r' be two rules over Σ. The distance between r and r' is $d(r,r') = |H(r) \ominus H(r')| + |B(r) \ominus B(r')|$ where $A \ominus B = (A\backslash B) \cup (B \backslash A)$ is the usual symmetric difference. The size of a rule r is defined as $|r| = |H(r)| + |B(r)|$.*

Definition 21. *Let P_1 and P_2 be programs over Σ. The distance between P_1 and P_2 is $dist(P_1, P_2) = Min\{dist_m(P_1,P_2) : m$ is a mapping between P_1 and $P_2\}$, where a mapping between P_1 and P_2 is a partial injective function $m : P_1 \to P_2$ and $dist_m(P_1,P_2) = Sum\{d(r, m(r)) : m(r) \in P_2\} + Sum\{|r| : r \in (P_1 \backslash m^{-1}[P_2])\} + Sum\{|r| : r \in (P_2 \backslash m[P_1])\}$.*

The findings about the distance between $\mathsf{f}_{SP}(P,q)$ and $\mathsf{f}^{sem}_{SP}(P,\{q\})$ in (Berthold et al., 2019) directly translate to f_R and f^*_{SP}.

Proposition 17. *Let P be a program over Σ and $q \in \Sigma$. Then, for each $r \in \mathsf{f}^{sem}_R \cup \mathsf{f}^{sem}_{SP}$, we have that $|r| \geq |\Sigma|$.*

Proposition 18. *Let P be a program over Σ and $q \in \Sigma$. Then,*

- $dist(P, \mathsf{f}^*_{SP}(P,\{q\})) \leq (|\mathsf{f}^{sem}_{SP}(P,\{q\})| + |P|) \times 2|\Sigma|$;
- $dist(P, \mathsf{f}^{sem}_{SP}(P,\{q\})) \geq (|\mathsf{f}^{sem}_{SP}(P,\{q\})| - |P|) \times |\Sigma|$;
- $dist(P, \mathsf{f}_R(P,\{q\})) \leq (|\mathsf{f}^{sem}_R(P,\{q\})| + |P|) \times 2|\Sigma|$;
- $dist(P, \mathsf{f}^{sem}_R(P,\{q\})) \geq (|\mathsf{f}^{sem}_R(P,\{q\})| - |P|) \times |\Sigma|$.

Proposition 19. *Let P be a program over Σ and $q \in \Sigma$.*

- *For each rule $r \in \mathsf{f}_{SP}(P,\{q\})$ there are at least 2^D rules in $\mathsf{f}^{sem}_{SP}(P,\{q\}$, with $D = Min(|H(r)|, |\Sigma\backslash\Sigma(r)|)$.*
- *For each rule $r \in \mathsf{f}_R(P,\{q\})$ there are at least 2^D rules in $\mathsf{f}^{sem}_R(P,\{q\}$, with $D = Min(|H(r)|, |\Sigma\backslash\Sigma(r)|)$.*

The three propositions above are proved, by substituting f_{SP} with f_R, resp. f^*_{SP}, in the corresponding proofs in (Berthold et al., 2019)