The Role of Self-attacking Arguments in Characterizations of Equivalence Notions*

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Abstract

A special case of loops in argumentation are self-attacking arguments. While their role with respect to the ontological nature of argumentation is controversially discussed, their presence (or absence) in the abstract setting of Dung-style argumentation frameworks seems to be less crucial for semantics or fundamental properties. There are, however, a few exceptions where self-attacking arguments have essential influence. One such exception concerns characterizations of (strong) equivalence notions between argumentation frameworks. Different notions of equivalence have recently been proposed in the literature and several characterization results for different semantics have been obtained. In this paper, we will survey the current state of this research direction with a particular emphasis on the effect of (dis)allowing self-conflicting arguments. We also provide some novel results for stage, eager, and naive semantics in order to present a full classification of ten prominent semantics and four equivalence notions.

keywords: abstract argumentation, equivalence relations.

1 Introduction

In the last two decades argumentation theory has received considerable interest in the AI-community (a basic overview is given in [36]). There are two main directions within this research field. *Deductive argumentation* (a comprehensive overview can be found in [14]) is concerned with constructing arguments from a given knowledge base and furthermore, determining their strength by applying a reasonable defined notion of attack. *Abstract argumentation*, on the other hand, studies the resolution of conflicts among arguments without considering their internal structure. The most known and extensively studied abstract system

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is the one proposed by Dung in [20], where digraphs (usually called abstract argumentation frameworks – AFs, for short) treat arguments and attacks as undefined primitives represented by vertices or edges, respectively. Moreover, a wide range of semantics for AFs is provided nowadays. The combined view on deductive and abstract argumentation is often referred to as instantiation-based argumentation, see for instance [18]. An issue often raised in the community is that investigations which entirely focus on abstract argumentation (and ignore the link to deductive argumentation) are of little use, since the obtained results might be meaningless when AFs are instantiated from knowledge bases. Nonetheless, studies on the pure abstract layer often help to gain new insight (in particular, since the logical contents of single arguments can be omitted) even in the instantiation-based context, as long as they are correctly interpreted in the big picture.

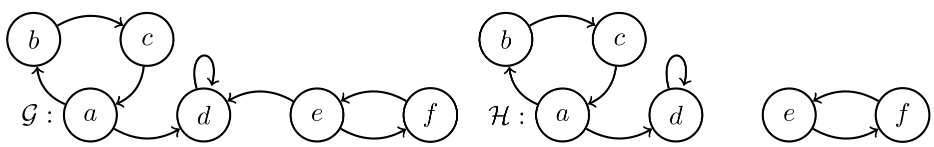
Self-attacking arguments are an example of a concept, where the relation between the abstract and general layer seems evident (if a constructed argument for whatever reason attacks itself, it is natural to represent this as a self-loop in the abstract framework). However, self-attacking arguments do not show up in all settings. For instance, in classical logic-based frameworks they do not occur (cf. [13, Theorem 4.13]) at all, while other argumentation systems like ASPIC [35] or a simple formalism due to Caminada [16] allow such self-defeating arguments. It is thus valuable to understand the influence of self-loops in the abstract domain, since this allows to make certain assumptions whenever we know under which instantiation-schema abstract frameworks are generated from.

Within the abstract setting, self-attacking arguments seem less controversial. In some papers they are even tacitly omitted. The reason is that for many problems, their presence/absence does not play a crucial role. However, there are important exceptions: First, as thoroughly discussed by Baroni and Giacomin [5] — similar issues have been raised in [15] — the presence of self-loops can lead to *unintuitive extensions* when AFs are evaluated (an observation they have used to define new semantics which treat odd-cycles – and thus self-loops – in a special manner); second, self-looping arguments turned out to be a comfortable tool when "artificial" arguments have to be added to AFs in order to spread a computation of a semantics to sub-frameworks (*splitting results*, see e.g. [6, 12, 11]), simulate a certain relation between arguments (*meta-argumentation*, see e.g. [33, 38]) or mimic different semantics (*intertranslatability*, see e.g. [25, 26]). Third, the absence of self-loops might decrease *complexity* in decision problem (examples where hardness actually depends on self-loops are a Π_2^P-hardness result [24, Proposition 11] and a P-hardness result [22, Theorem 25]). Finally, self-loops turned out to be the central object when it comes to characterize different *notions of equivalence*. This is also the topic of this paper.

In general, equivalence tells us whether two syntactically different object represent the same information - which is relevant, for instance, for simplification issues. In monotonic formalisms, there is usually one standard notion of equivalence which serves this purpose as well as the purpose of "equivalence for replacement", where one asks whether a replacement of equivalent objects within a larger one, does not change the meaning of the latter. For nonmonotonic formalisms these two purposes require, in general, different notions of equivalence, see e.g. [31]. Also abstract argumentation frameworks are inherently non-monotonic, thus standard equivalence (in terms of extensions) does not imply equivalence for replacement.

Example 1. *The AFs $\mathcal{F}_1$ and $\mathcal{F}_2$ possess the unique semi-stable*[1] *extension $\{e\}$.*

The AF $\mathcal{H}$ syntactically results by replacing the sub-framework $\mathcal{F}_1$ of $\mathcal{G}$ with $\mathcal{F}_2$. One may verify that $\mathcal{G}$ has a unique semi-stable extension $\{e\}$, while $\mathcal{H}$ possess two such extensions, $\{e\}$ and $\{f\}$.



This line of research was initiated by Oikarinen and Woltran [34] who introduced the notion of *strong equivalence* for abstract AFs as follows. Two AFs $\mathcal{F}$ and $\mathcal{G}$ are strongly equivalent if for any AF $\mathcal{H}$, $\mathcal{F}$ conjoined with $\mathcal{H}$ and $\mathcal{G}$ conjoined with $\mathcal{H}$ possess the same extensions. The difference between standard and strong equivalence also suggests to study equivalence notions in-between them. The weaker notion of *local equivalence* was also introduced in [34] adapting the notion of strong equivalence by restricting $\mathcal{H}$ to existing arguments (i.e. only arguments from $\mathcal{F}$ or $\mathcal{G}$ are allowed to occur in $\mathcal{H}$). Baumann [7] then introduced further notions, among them *normal expansion equivalence* and *strong expansion equivalence*. The former disallows in $\mathcal{H}$ the addition of attacks between arguments already given in $\mathcal{F}$ and $\mathcal{G}$. The latter does not permit attacks originating from existing arguments. Their relevance is thoroughly discussed in [7], we only mention here that in dynamic settings it seems natural that the behavior of already given arguments should not change when new arguments are raised. Further notions of equivalence, which we will not discuss in this paper, are weak expansion equivalence [6], symmetric equivalence [27] and minimal change equivalence [8].

As a valuable tool for deciding strong equivalence, Oikarinen and Woltran [34] introduced the notion of a *kernel* of an AF. Informally speaking, kernels are frameworks without redundant attacks. The significant insight of [34] was that strong equivalence for most semantics is exactly captured by syntactical equivalence of suitably chosen notions of kernels. All kernels found so far differed only in case where self-loops are permitted; in other words, if self-loops are omitted

[1] See section 2 for the exact definition of semi-stable extensions.

it turns out that each framework is its own kernel (a related observation for graph problems in general was given independently at this time in [32]).

As we already have mentioned, numerous semantics are available for abstract argumentation frameworks among those already defined by Dung [20], namely admissible, stable, complete, grounded, and preferred semantics. To overcome the problem of non-existence of stable extensions, stage and semi-stable semantics [19, 37] were suggested. Another approach for semantics is driven by the motivation that unique-status semantics are important but the grounded semantics (the only unique-statues semantics originally proposed) is too cautious. Ideal and eager semantics [17, 21] follow this idea. Finally, we consider here also the so-called naive semantics. Kernel-based results for strong equivalence for these ten semantics have been given in [34, 27]. Also local equivalence has been analyzed but not all characterizations given so far admit a kernel. Normal and respectively strong expansion equivalence have been characterized in [7], but without taking eager, stage, and naive semantics into account.

Our main contributions are thus the following:

- We survey the existing literature on four equivalence notions (strong, local, normal expansion, strong expansion) for the ten semantics mentioned above, thereby
- clarifying open questions concerning eager, stage and naive semantics.
- Moreover, we provide an alternative characterization for local equivalence with respect to stage semantics in terms of a kernel.
- Finally, we discuss the collection of results in the light of self-attacking arguments and the role they play in these characterizations.

The organization of the paper is as follows. Section 2 reviews the necessary definitions in abstract argumentation including several argumentation semantics, notions of expansions as well as equivalence, and the concept of a kernel. In Section 3 we present our new results, where we first focus on stage semantics and then continue to close open gaps in terms of naive and eager semantics. Section 4 then presents the overview of obtained characterizations; we discuss these results in detail and clarify the role of self-loops. Finally, in Section 5 we conclude and give pointers to future work.

2 Preliminaries

An *argumentation framework (AF)* is a pair $\mathcal{F} = (A, R)$, where A is a set whose elements are called *arguments* and $R \subseteq A \times A$ a binary relation, called the *attack relation*. In this paper we restrict ourselves to finite AFs including the empty framework. If $(a, b) \in R$ holds we say that *a attacks* b, or b is *defeated* by a in $\mathcal{F}$. Furthermore, we will slightly abuse notations, and write $(A, b) \in R$ for $\exists a \in A : (a, b) \in R$; likewise we use $(b, A) \in R$ and $(A, A') \in R$. An argument $a \in A$ is *defended* by a set $A' \subseteq A$ in $\mathcal{F}$ if for each $b \in A$ with $(b, a) \in R$,

$(A', b) \in R$. A set $A' \subseteq A$ is called *conflict-free* in $\mathcal{F}$ if there are no arguments $a, b \in A'$ such that a attacks b. The set of all conflict-free sets of an AF $\mathcal{F}$ is denoted by $cf(\mathcal{F})$. For an AF $\mathcal{F} = (B, S)$ we use $A(\mathcal{F})$ to refer to B and $R(\mathcal{F})$ to refer to S. Finally, we introduce the union for two AFs $\mathcal{F}$ and $\mathcal{G}$ as expected, namely $\mathcal{F} \cup \mathcal{G} = (A(\mathcal{F}) \cup A(\mathcal{G}), R(\mathcal{F}) \cup R(\mathcal{G}))$.

2.1 Semantics

A semantics σ is a function which assigns to any AF $\mathcal{F}$ a set of sets of arguments denoted by $\mathcal{E}_\sigma(\mathcal{F})$. Each one of them, a so-called σ-*extension*, is considered to be acceptable w.r.t. $\mathcal{F}$. Numerous semantics are available for abstract argumentation frameworks among those already defined by Dung [20], namely stable, admissible, preferred, complete and grounded semantics. Further semantics we consider here are semi-stable and stage semantics [19, 37] as well as the unique-status semantics ideal [21] and eager [17]. Finally, we consider here also the so-called naive semantics. For recent overviews over different semantics for abstract argumentation, we refer the reader to [4, 2].

Definition 1. *Let $\mathcal{F} = (A, R)$ be an AF and $E \subseteq A$. E is a*

1. *stable extension of $\mathcal{F}$ ($E \in \mathcal{E}_{stb}(\mathcal{F})$) iff*
   $E \in cf(\mathcal{F})$ and for every $a \in A \setminus E$, $(E, a) \in R$ holds,

2. *admissible set of $\mathcal{F}$ ($E \in \mathcal{E}_{ad}(\mathcal{F})$) iff*
   $E \in cf(\mathcal{F})$ and each $a \in E$ is defended by E in $\mathcal{A}$,

3. *preferred extension of $\mathcal{F}$ ($E \in \mathcal{E}_{pr}(\mathcal{F})$) iff*
   $E \in \mathcal{E}_{ad}(\mathcal{F})$ and for each $E' \in \mathcal{E}_{ad}(\mathcal{F})$, $E \not\subset E'$ holds,

4. *complete extension of $\mathcal{F}$ ($E \in \mathcal{E}_{co}(\mathcal{F})$) iff*
   $E \in \mathcal{E}_{ad}(\mathcal{F})$ and for each $a \in A$ defended by E in $\mathcal{F}$, $a \in E$ holds,

5. *grounded extension of $\mathcal{F}$ ($E \in \mathcal{E}_{gr}(\mathcal{F})$) iff*
   $E \in \mathcal{E}_{co}(\mathcal{F})$ and for each $E' \in \mathcal{E}_{co}(\mathcal{F})$, $E' \not\subset E$ holds,

6. *semi-stable extension of $\mathcal{F}$ ($E \in \mathcal{E}_{ss}(\mathcal{F})$) iff*
   $E \in \mathcal{E}_{ad}(\mathcal{F})$ and for each $E' \in \mathcal{E}_{ad}(\mathcal{F})$, $R^+_\mathcal{F}(E) \not\subset R^+_\mathcal{F}(E')$ holds, where $R^+_\mathcal{F}(E) = E \cup \{b \mid (a, b) \in R, a \in E\}$,

7. *stage extension of $\mathcal{F}$ ($E \in \mathcal{E}_{stg}(\mathcal{F})$) iff*
   $E \in cf(\mathcal{F})$ and for each $E' \in cf(\mathcal{F})$, $R^+_\mathcal{F}(E) \not\subset R^+_\mathcal{F}(E')$ holds,

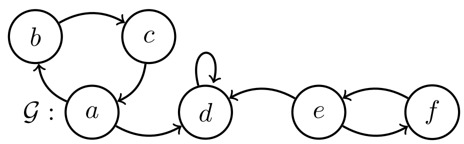
8. *ideal extension of $\mathcal{F}$ ($E \in \mathcal{E}_{id}(\mathcal{F})$) iff*
   $E \in \mathcal{E}_{ad}(\mathcal{F})$, $E \subseteq \bigcap_{P \in \mathcal{E}_{pr}(\mathcal{F})} P$ and for each $E' \in \mathcal{E}_{ad}(\mathcal{F})$ with the property $E' \subseteq \bigcap_{P \in \mathcal{E}_{pr}(\mathcal{F})} P$, $E \not\subset E'$ holds,

9. *eager extension of $\mathcal{F}$ ($E \in \mathcal{E}_{eg}(\mathcal{F})$) iff*
   $E \in \mathcal{E}_{ad}(\mathcal{F})$, $E \subseteq \bigcap_{P \in \mathcal{E}_{ss}(\mathcal{F})} P$ and for each $E' \in \mathcal{E}_{ad}(\mathcal{F})$ with the property $E' \subseteq \bigcap_{P \in \mathcal{E}_{ss}(\mathcal{F})} P$, $E \not\subset E'$ holds,

10. *naive extension of $\mathcal{F}$ ($E \in \mathcal{E}_{na}(\mathcal{F})$) iff*
  $E \in cf(\mathcal{F})$ and for each $E' \in cf(\mathcal{F})$, $E \not\subset E'$ holds.

The following example illustrates the semantics defined above.

Example 2 (Example 1 cont.)**.** *Consider again the AF $\mathcal{G}$.*



The listed sets in the first column represent the conflict-free sets of $\mathcal{G}$. The entry $\times$ in row C and column σ indicates that the conflict-free set C is a σ-extension of $\mathcal{G}$.

	stb	*ss*	*stg*	*pr*	*ad*	*co*	*gr*	*id*	*eg*	*na*
$\emptyset$					$\times$	$\times$	$\times$	$\times$		
$\{e\}$		$\times$		$\times$	$\times$	$\times$			$\times$	
$\{f\}$				$\times$	$\times$	$\times$				
$\{a, e\}$			$\times$							$\times$
$\{b, e\}$			$\times$							$\times$
$\{c, e\}$			$\times$							$\times$
$\{a, f\}$			$\times$							$\times$
$\{b, f\}$										$\times$
$\{c, f\}$										$\times$

There are several relations between the considered semantics. First of all, for any AF $\mathcal{F}$,

$$\mathcal{E}_{stb}(\mathcal{F}) \subseteq \mathcal{E}_{ss}(\mathcal{F}) \subseteq \mathcal{E}_{pr}(\mathcal{F}) \subseteq \mathcal{E}_{co}(\mathcal{F}) \subseteq \mathcal{E}_{ad}(\mathcal{F}) \subseteq cf(\mathcal{F})$$

and furthermore,

$$\mathcal{E}_{stb}(\mathcal{F}) \subseteq \mathcal{E}_{stg}(\mathcal{F}) \subseteq \mathcal{E}_{na}(\mathcal{F}) \subseteq cf(\mathcal{F}).$$

The table above shows that stable extensions do not necessarily exist. However, the other semantics in question warrant the existence of at least one extension in case of finite AFs[2]. Grounded, ideal and eager semantics possess exactly one extension for any AF $\mathcal{F}$. Furthermore, we have $G \subseteq I \subseteq E$ if G, I and E are the unique grounded, ideal and eager extensions of an AF $\mathcal{F}$.

2.2 Expansions and Corresponding Equivalence Notions

In a typical argumentation scenario arguments and counterarguments are put forward alternately. Thus, argumentation can be seen as an inherently dynamic process. Depending on the way AFs may be manipulated within this process, different notions of equivalence have been defined in order to decide whether two AFs behave the same in such scenarios.

Oikarinen and Woltran [34] studied equivalence w.r.t. arbitrary expansions. This means, any additions of arguments and/or attacks (even between existing arguments) are allowed. Baumann [7] relaxed this criterion and characterized equivalence w.r.t. *normal* and *strong expansions* firstly introduced in [9]. These are expansions where the attack relation between existing arguments remains unchanged. Such kind of expansions occur, for instance, in the overall instantiation process if a new piece of information is added to the underlying knowledge base (see [29]). *Local expansions* [34] are an orthogonal concept to normal expansions where no new arguments are raised, but the attack relation can be augmented.

Here are the formal definitions.

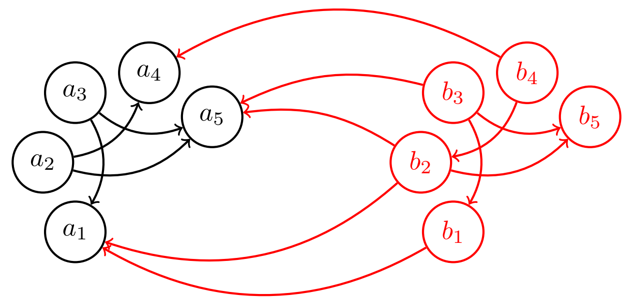
Definition 2. *An AF $\mathcal{F}^*$ is an expansion of an AF $\mathcal{F} = (A, R)$ (in symbols, $\mathcal{F} \preceq_E \mathcal{F}^*$) iff $\mathcal{F}^* = (A \cup A^*, R \cup R^*)$ where $A^* \cap A = R^* \cap R = \emptyset$. An expansion is called*

1. *normal ($\mathcal{F} \preceq_N \mathcal{F}^*$) iff $\forall ab \ \ ((a, b) \in R^* \rightarrow a \in A^* \vee b \in A^*)$,*
2. *strong ($\mathcal{F} \preceq_S \mathcal{F}^*$) iff $\mathcal{F} \preceq_N \mathcal{F}^*$ and $\forall ab \ \ ((a, b) \in R^* \rightarrow \neg(a \in A \wedge b \in A^*))$,*
3. *local ($\mathcal{F} \preceq_L \mathcal{F}^*$) iff $A^* = \emptyset$.*

For short, normal expansions add new arguments and possibly new attacks which involve at least one of the fresh arguments. Strong expansions are normal and only add arguments which are not attacked by existing arguments. Finally, local expansions do not introduce any new arguments but possibly new attacks among the old arguments.

Example 3 (An illustration of a strong expansion)**.** *The arguments and attacks marked with red represent the added information.*

[2]We mention that stage and semi-stable semantics do not serve as an unrestricted alternative for stable semantics since the existence of extensions is not guaranteed in case of infinite AFs [39, 37].



Now we define the corresponding notions of equivalence. For the sake of clarity and comprehensibility we use *expansion equivalence* instead of *strong equivalence* (the term originally coined in [34]) to indicate that arbitrary expansions are allowed.

Definition 3. *Given a semantics σ. Two AFs $\mathcal{F}$ and $\mathcal{G}$ are*

1. *expansion equivalent w.r.t. σ ($\mathcal{F} \equiv_E^\sigma \mathcal{G}$) iff for each AF $\mathcal{H}$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,*

2. *normal expansion equivalent w.r.t. σ ($\mathcal{F} \equiv_N^\sigma \mathcal{G}$) iff for each AF $\mathcal{H}$, s.t. $\mathcal{F} \preceq_N \mathcal{F} \cup \mathcal{H}$ and $\mathcal{G} \preceq_N \mathcal{G} \cup \mathcal{H}$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,*

3. *strong expansion equivalent w.r.t. σ ($\mathcal{F} \equiv_S^\sigma \mathcal{G}$) iff for each AF $\mathcal{H}$, s.t. $\mathcal{F} \preceq_S \mathcal{F} \cup \mathcal{H}$ and $\mathcal{G} \preceq_S \mathcal{G} \cup \mathcal{H}$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,*

4. *local expansion equivalent w.r.t. σ ($\mathcal{F} \equiv_L^\sigma \mathcal{G}$) iff for each AF $\mathcal{H}$, s.t. $A(\mathcal{H}) \subseteq A(\mathcal{F} \cup \mathcal{G})$, $\mathcal{F} \cup \mathcal{H} \equiv^\sigma \mathcal{G} \cup \mathcal{H}$ holds,*

5. *standard equivalent w.r.t. σ ($\mathcal{F} \equiv^\sigma \mathcal{G}$) iff $\mathcal{F}$ and $\mathcal{G}$ possess the same extensions under σ, i.e. $\mathcal{E}_\sigma(\mathcal{F}) = \mathcal{E}_\sigma(\mathcal{G})$ holds.*

The different notions of equivalence guarantee mutual replaceability for the corresponding dynamic scenarios. Furthermore, the subset relations of the underlying classes of expansions imply relations of the corresponding notions of equivalence. Consider therefore the following proposition or its more eye-catching graphical representation below.

Proposition 1. *For any AFs $\mathcal{F}$, $\mathcal{G}$, and* ***any*** *semantics σ the following holds:*

1. $\mathcal{F} \equiv_E^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv_N^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv_S^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv^\sigma \mathcal{G}$

2. $\mathcal{F} \equiv_E^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv_L^\sigma \mathcal{G} \Rightarrow \mathcal{F} \equiv^\sigma \mathcal{G}$

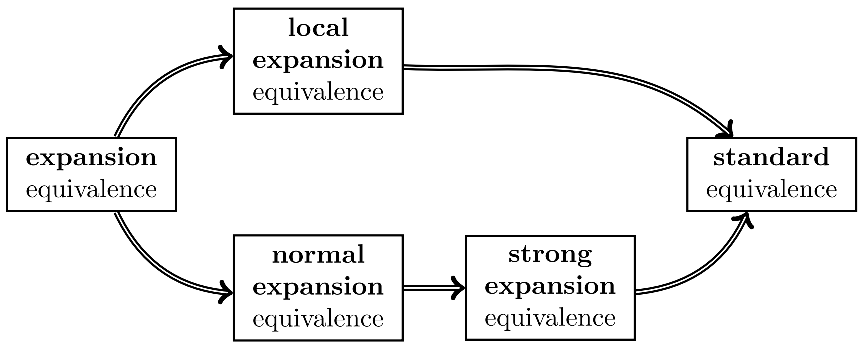


Figure 1: Relations between Equivalence Notions

In general the converse directions do not necessarily hold. It is one aim of this paper to underline that for the considered semantics several converse directions hold. In particular, normal expansion equivalence coincides with expansion equivalence for each considered semantics.

2.3 Deciding Equivalence via Kernels

One main question is how to decide whether two AFs are equivalent w.r.t. a certain kind of expansion. It is one main result that in most cases equivalence can be decided via so-called *kernels* [34, 7]. A kernel of an AF $\mathcal{F}$ is itself an AF obtained from $\mathcal{F}$ by deleting certain attacks based on the actual structure of the given AF but without computing extensions or any related semantical objects. This makes kernels a purely syntactical — and thus efficiently computable — concept. A kernel characterizes an equivalence relation as follows: two AFs are equivalent if and only if their corresponding kernels are identical. In other words, the kernel detects the redundant attacks w.r.t. a certain kind of expansion and semantics. Consider, for instance, stable extensions and expansion equivalence. Intuitively, an attack (a, b) in an AF $\mathcal{F}$ is redundant for this semantics, only if a is guaranteed to be excluded from any stable extension (no matter how $\mathcal{F}$ is expanded). The only arguments satisfying this condition are the ones which are self-attacking; any other argument b which is not attacking itself can be enforced to be in an extension by adding attack (b, c), for each argument c ($c \neq b$) in $\mathcal{F}$. Thus, the only redundant attacks which have to be deleted to obtain a *stb*-kernel are those (a, b) where also (a, a) holds.

The following definition provides an exhaustive overview of all existing and characterizing kernels. The so-called *σ-kernels* were introduced in [34] to characterize expansion equivalence. In [7] it was shown that in case of strong expansion equivalence other characterizations are required. Therefore so-called *σ-*-kernels* were introduced which allow more deletions than their counterparts for expansion equivalence.

Definition 4. *Given an AF* $\mathcal{F} = (A, R)$ *and a semantics* σ. *We define σ-kernels* $\mathcal{F}^{k(\sigma)} = (A, R^{k(\sigma)})$ *(resp. σ-*-kernels* $\mathcal{F}^{k^*(\sigma)} = (A, R^{k^*(\sigma)})$*) whereby*

1. $R^{k(stb)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R\}$,

2. $R^{k(ad)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R, \{(b,a),(b,b)\} \cap R \neq \emptyset\}$,

3. $R^{k^*(ad)} = R \setminus \{(a,b) \mid a \neq b, ((a,a) \in R \wedge \{(b,a),(b,b)\} \cap R \neq \emptyset) \vee$
   $((b,b) \in R \wedge \forall c\ ((b,c) \in R \to \{(a,c),(c,a),(c,c),(c,b)\} \cap R \neq \emptyset))\}$,

4. $R^{k(gr)} = R \setminus \{(a,b) \mid a \neq b, (b,b) \in R, \{(a,a),(b,a)\} \cap R \neq \emptyset\}$,

5. $R^{k^*(gr)} = R \setminus \{(a,b) \mid a \neq b, ((b,b) \in R \wedge \{(a,a),(b,a)\} \cap R \neq \emptyset) \vee$
   $((b,b) \in R \wedge \forall c\ ((b,c) \in R \to \{(a,c),(c,a),(c,c)\} \cap R \neq \emptyset))\}$,

6. $R^{k(co)} = R \setminus \{(a,b) \mid a \neq b, (a,a),(b,b) \in R\}$,

7. $R^{k^*(co)} = R \setminus \{(a,b) \mid a \neq b, ((a,a),(b,b) \in R) \vee$
   $((b,b) \in R \wedge (b,a) \notin R \wedge \forall c\ ((b,c) \in R \to \{(a,c),(c,a),(c,c)\} \cap R \neq \emptyset))\}$.

Observe that a framework $\mathcal{F}$ and its corresponding kernel share the same arguments. Furthermore, any attack contained in the kernel stems from the initial framework $\mathcal{F}$.

We already gave the intuition for *stb*-kernels above. Note that the other kernels typically allow less removals of attacks. The reason is that compared to stable extensions, the notion of defense plays a role. Hence, an attack (a,b) where (a,a) holds still can be necessary for defending b against a (suppose there is additional attack (c,a)). However, if also b is self-attacking or b is defending itself against a, then (a,b) becomes redundant. This is exactly what the definition of an *ad*-kernel captures. The intuition behind the *gr*- and *co*-kernels are similar. Furthermore, the difference between a σ-kernel and its corresponding σ-*-kernel is usually a further disjunct allowing for more deletions. This is due to the fact that σ-*-kernels were introduced to characterize strong expansion equivalence [7]. The special feature of strong expansions is that a former attack between old arguments will never become a counterattack to an added attack. In other words, a former attack does not play a role with respect to being a potential defender of an added argument. Consequently, in contrast to arbitrary expansions where such attacks might be relevant, we may delete them without changing the behavior with respect to further evaluations.

The following table summarizes the state of the art. The entry "$k(\sigma)$" in row X and column τ indicates that the σ-kernel characterizes $\equiv^{\tau}_{X}$. The entry "$[m]_n$" is a shorthand for: The characterization problem is already solved in [m] (Theorem n), but there is no kernel provided so far. The entry "?" represents an open problem which will be solved in this paper.

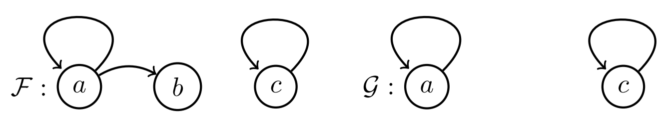
	stg	*stb*	*ss*	*eg*	*ad*	*pr*	*id*	*gr*	*co*	*na*
L	$[27]_6$	$[34]_9$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$[34]_{10}$	$[34]_{11}$	$[28]_{5.13}$
E	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$[28]_{5.13}$
N	?	$k(stb)$	$k(ad)$	?	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	?
S	?	$k(stb)$	$k(ad)$	?	$k^*(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(gr)$	$k^*(co)$	?

Figure 2: State of the Art

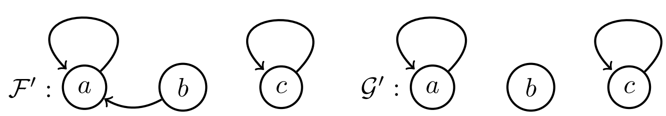
3 Characterizing further Equivalence Notions for Stage, Naive and Eager Semantics

In this section we will first show that stage and stable semantics behave in the same manner w.r.t. the characterization of expansion, normal expansion and strong expansion equivalence. This means, all mentioned equivalence notions are captured by the stable-kernel. In case of local expansion equivalence we observe a certain difference. Consider therefore the following example.

Example 4. *The AFs $\mathcal{F}$ and $\mathcal{G}$ are local expansion equivalent w.r.t. stable semantics. This can be seen by checking that $\mathcal{E}_{stb}(\mathcal{F} \cup \mathcal{H}) = \mathcal{E}_{stb}(\mathcal{G} \cup \mathcal{H})$ for any $\mathcal{H}$, s.t. $A(\mathcal{H}) \subseteq \{a, b, c\}$.*[3]



Obviously, $\mathcal{F}$ and $\mathcal{G}$ are not local expansion equivalent w.r.t. stage semantics because $\{\{b\}\} = \mathcal{E}_{stg}(\mathcal{F}) \neq \mathcal{E}_{stg}(\mathcal{G}) = \{\emptyset\}$. Let us consider now two further AFs $\mathcal{F}'$ and $\mathcal{G}'$ slightly different to $\mathcal{F}$ and $\mathcal{G}$.



[3] More precisely, local expansion equivalence is due to the fact that $\mathcal{E}_{stb}(\mathcal{F}) = \mathcal{E}_{stb}(\mathcal{G}) = \emptyset$, $A(\mathcal{F}) \setminus A(\mathcal{G}) = \{b\}$, $\{(b,b),(b,c)\} \cap R(\mathcal{F} \cup \mathcal{G}) = \emptyset$ and $(c,c) \in R(\mathcal{F} \cup \mathcal{G})$ (compare [34, Theorem 9]).

We state that $\mathcal{F}'$ and $\mathcal{G}'$ are not local expansion equivalent w.r.t. stable semantics. One possible scenario which makes the predicted different behaviour explicit is the following where $\mathcal{H} = (\{b, c\}, \{(b, c)\})$. Obviously, $\{\{b\}\} = \mathcal{E}_{stb}(\mathcal{F}' \cup \mathcal{H}) \neq \mathcal{E}_{stb}(\mathcal{G}' \cup \mathcal{H}) = \emptyset$. It is one main result of this section showing that AFs like $\mathcal{F}'$ and $\mathcal{G}'$ are local expansion equivalent w.r.t. stage semantics because they possess identical stage--kernels.*[4]

We then proceed with naive semantics and show that expansion, normal expansion, strong expansion as well as local expansion equivalence coincide, thus generalizing the results from [28]. In particular, possessing the same naive extensions and sharing the same arguments is sufficient and necessary for all aforementioned equivalence notions.

Finally, we will show that in case of eager semantics all considered equivalence relations are characterized by the admissible-kernel. In this regard, eager semantics is closer to semi-stable semantics than ideal semantics.

3.1 Normal and Strong Expansion Equivalence w.r.t. Stage Semantics

We will show now that strong expansion equivalence w.r.t. stage semantics can be decided by comparing their corresponding stable-kernels. Remember that the stable-kernel of an AF $\mathcal{F} = (A, R)$ is $\mathcal{F}^{k(stb)} = (A, R \backslash \{(a, b) \mid a \neq b, (a, a) \in R\})$.

Theorem 1. *For any AFs $\mathcal{F}$, $\mathcal{G}$ and $\Phi \in \{N, S\}$,*

$$\mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)} \Leftrightarrow \mathcal{F} \equiv^{stg}_{\Phi} \mathcal{G}.$$

Proof. Since $\mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)} \Rightarrow \mathcal{F} \equiv^{stg}_{E} \mathcal{G} \Rightarrow \mathcal{F} \equiv^{stg}_{N} \mathcal{G} \Rightarrow \mathcal{F} \equiv^{stg}_{S} \mathcal{G}$ is by [28, Theorem 5.11] and Proposition 1, we only have to show that $\mathcal{F} \equiv^{stg}_{S} \mathcal{G} \Rightarrow \mathcal{F}^{k(stb)} = \mathcal{G}^{k(stb)}$ holds. We prove this implication by contraposition.

Suppose $\mathcal{F}^{k(stb)} \neq \mathcal{G}^{k(stb)}$. **1**^{stb} **case**: Consider $A\left(\mathcal{F}^{k(stb)}\right) \neq A\left(\mathcal{G}^{k(stb)}\right)$. Consequently, w.l.o.g. there exists an argument $a \in A(\mathcal{F}) \setminus A(\mathcal{G})$. Let c be a fresh argument and $B = A(\mathcal{F} \cup \mathcal{G}) \setminus \{a\}$. We define

$$\mathcal{H} = \left(B \cup \{c\}, \{(c, c') \,|c' \in B\}\right).$$

If a is contained in some $E \in \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{H})$, then $E \notin \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{H})$ since $a \notin A(\mathcal{G} \cup \mathcal{H})$ was supposed. If not, consider $\mathcal{H}' = \mathcal{H} \cup (\{a\}, \emptyset)$. Then, $E = \{a, c\}$ is a stable extension in $\mathcal{G} \cup \mathcal{H}'$ and therefore, $E \in \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{H}')$. Furthermore, $E \notin \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{H}')$ since $\mathcal{F} \cup \mathcal{H}' = \mathcal{F} \cup \mathcal{H}$ holds. Consequently, $\mathcal{F} \not\equiv^{stg}_{S} \mathcal{G}$ is shown.

2nd **case**: Consider $A\left(\mathcal{F}^{k(stb)}\right) = A\left(\mathcal{G}^{k(stb)}\right)$ and $R\left(\mathcal{F}^{k(stb)}\right) \neq R\left(\mathcal{G}^{k(stb)}\right)$. Hence, there are $a, b \in A(\mathcal{F})$, s.t. $(a, b) \in R\left(\mathcal{F}^{k(stb)}\right) \backslash R\left(\mathcal{G}^{k(stb)}\right)$. Let c be a fresh argument. We define

$$\mathcal{I} = \left(A(\mathcal{F}) \cup \{c\}, \{(c, c') \,|c' \in A(\mathcal{F}) \backslash \{a, b\}\}\right).$$

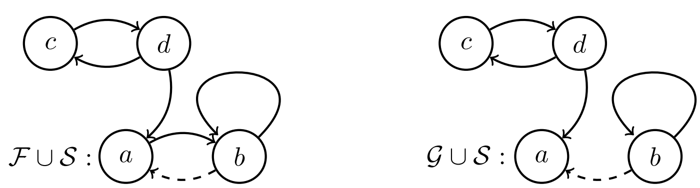
[4]Note that a characterization theorem for local expansion equivalence w.r.t. stage semantics is already given in [27, Theorem 6]. In this paper we will present an appropriate kernel, the so-called stage-*-kernel.

Case **2.1:** Let $a = b$, therefore $(a, a) \in R\left(\mathcal{F}^{k(stb)}\right) \setminus R\left(\mathcal{G}^{k(stb)}\right)$ and consequently, $(a, a) \in R(\mathcal{F}) \setminus R(\mathcal{G})$ because self-loops remain the same after applying the kernel-operator. Obviously, $\mathcal{E}_{stb}(\mathcal{G} \cup \mathcal{I}) = \{\{a, c\}\}$ and thus, $\{a, c\} \in \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{I})$. On the other hand, $\{a, c\} \notin cf(\mathcal{F} \cup \mathcal{I})$ and therefore, $\{a, c\} \notin \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{I})$. This means, $\mathcal{F} \not\equiv_S^{stg} \mathcal{G}$ is shown. Thus from now on we assume that $R\left(\mathcal{F}^{k(stb)}\right)$ and $R\left(\mathcal{G}^{k(stb)}\right)$ contain the same self-loops.

Case **2.2:** Let $a \neq b$. Since $(a, b) \in R\left(\mathcal{F}^{k(stb)}\right) \setminus R\left(\mathcal{G}^{k(stb)}\right)$, it follows that $(a, b) \in R(\mathcal{F})$, $(a, a) \notin R(\mathcal{F})$, consequently $(a, a) \notin R(\mathcal{G})$ and $(a, b) \notin R(\mathcal{G})$. At first, we assume $(b, b) \notin R(\mathcal{F})$. Thus, $(b, b) \notin R(\mathcal{G})$. In any case $\{a, c\} \in \mathcal{E}_{stb}(\mathcal{F} \cup \mathcal{I})$ and consequently, $\{a, c\} \in \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{I})$. On the other hand, we state $\{a, c\} \notin \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{I})$ because $\{\{a, b, c\}\} = \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{I})$ if $(b, a) \notin R(\mathcal{G})$ and $\{\{b, c\}\} = \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{I})$ if $(b, a) \in R(\mathcal{G})$. Second, let $(b, b) \in R(\mathcal{F})$. Consequently, $(b, b) \in R(\mathcal{G})$. In contrast to the other cases the AF $\mathcal{I}$ does not enforce different stage extensions. In particular, $\mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{I}) = \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{I}) = \{\{a, c\}\}$ independently of the occurrence of the attack (b, a) in $\mathcal{F}$ or $\mathcal{G}$. We define

$$\mathcal{S} = (A(\mathcal{F}) \cup \{c, d\}, \{(e, f) \mid e \in \{c, d\} \wedge f \in A(\mathcal{F}) \setminus \{a, b\}\} \cup \{(c, d), (d, a), (d, c)\}).$$

For the purpose of illustration we omit further arguments different from a, b, c and d. Bear in mind that these arguments are attacked by c and d. The dashed arrows reflect the situation that (b, a) may or not be in $R(\mathcal{F})$ or $R(\mathcal{G})$.



Note that $b \notin R^+_{\mathcal{G} \cup \mathcal{S}}(E)$ for any set $E \in cf(\mathcal{G} \cup \mathcal{S})$. Since $R^+_{\mathcal{G} \cup \mathcal{S}}(\{d\}) = A(\mathcal{G} \cup \mathcal{S}) \setminus \{b\}$ we deduce $\{d\} \in \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{S})$. On the other hand, $R^+_{\mathcal{F} \cup \mathcal{S}}(\{d\}) = A(\mathcal{F} \cup \mathcal{S}) \setminus \{b\} \subset A(\mathcal{F} \cup \mathcal{S}) = R^+_{\mathcal{F} \cup \mathcal{S}}(\{a, c\})$. Consequently, $\{d\} \notin \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{S})$ is shown. Thus, $\mathcal{F} \not\equiv_S^{stg} \mathcal{G}$ concluding the proof. □

3.2 Local Expansion Equivalence w.r.t. Stage Semantics

We already know that strong equivalence implies local expansion equivalence for any semantics σ (compare Proposition 1). Furthermore, there are certain semantics where local expansion equivalence is even sufficient for strong equivalence. This is, for example, the case if we consider semi-stable semantics [34, Theorem 8]. In case of stage semantics an analogous result does not hold, i.e., for this semantics, local expansion equivalence is a properly weaker concept than expansion equivalence. We will see that the characterization of local expansion equivalence w.r.t. stage semantics can be done by comparing a newly introduced *stage-*-kernel* which is defined as follows.

Definition 5. *Given a an AF $\mathcal{F} = (A, R)$. We define the stage-*-kernel of $\mathcal{F}$*

as $\mathcal{F}^{k^*(stg)} = (A, R^{k^*(stg)})$ where

$$R^{k^*(stg)} = R \setminus \{(a,b) \mid a \neq b, (a,a) \in R \vee \forall c\ (c \neq a \rightarrow (c,c) \in R)\}.$$

The first disjunct allows the deletion of an attack (a,b) if a is self-attacking. These attacks are even redundant w.r.t. arbitrary expansions (compare [28, Theorem 5.11]). The second disjunct reflects the intuition that an attack (a,b) becomes irrelevant w.r.t. local expansions and the evaluation given by stage semantics if all arguments different from a are self-attacking. Given the latter scenario we observe that only two sets may be a stage extension, namely $\emptyset$ or $\{a\}$. In particular, the conflict-freeness of $\{a\}$ is sufficient for being the unique stage extension.

At first we will prove two technical lemmata paving the way for the main theorem showing that the syntactical equivalence of stage-*-kernels characterizes local expansion equivalence between two AFs w.r.t. stage semantics.

Lemma 1. *If $\mathcal{F}^{k^*(stg)} = \mathcal{G}^{k^*(stg)}$, then $(\mathcal{F} \cup \mathcal{H})^{k^*(stg)} = (\mathcal{G} \cup \mathcal{H})^{k^*(stg)}$ for any AF $\mathcal{H}$, s.t. $A(\mathcal{H}) \subseteq A(\mathcal{F} \cup \mathcal{G})$.*

Proof. First notice that the assumption $\mathcal{F}^{k^*(stg)} = \mathcal{G}^{k^*(stg)}$ implies that $R(\mathcal{F})$ and $R(\mathcal{G})$ contain the same self-loops. Furthermore, $A(\mathcal{F}) = A(\mathcal{G})$ and thus, $A(\mathcal{F}) = A\left((\mathcal{F} \cup \mathcal{H})^{k^*(stg)}\right) = A\left((\mathcal{G} \cup \mathcal{H})^{k^*(stg)}\right)$. Hence, it suffices to show that $R\left((\mathcal{F} \cup \mathcal{H})^{k^*(stg)}\right) = R\left((\mathcal{G} \cup \mathcal{H})^{k^*(stg)}\right)$. Assume $(a,b) \in R\left((\mathcal{F} \cup \mathcal{H})^{k^*(stg)}\right) \setminus R\left((\mathcal{G} \cup \mathcal{H})^{k^*(stg)}\right)$. It suffices to consider $a \neq b$ since the sharing of the same self-loops of $(\mathcal{F} \cup \mathcal{H})^{k^*(stg)}$ and $(\mathcal{G} \cup \mathcal{H})^{k^*(stg)}$ is implied. We deduce $(a,b) \in R(\mathcal{F} \cup \mathcal{H})$ and furthermore, $(a,a) \notin R(\mathcal{F}), R(\mathcal{H})$ and thus, $(a,a) \notin R(\mathcal{G})$. Additionally, there exists an argument c, s.t. $c \neq a$ and $(c,c) \notin R(\mathcal{F} \cup \mathcal{H})$. Thus, $(c,c) \notin R(\mathcal{F}), R(\mathcal{H})$ and consequently, $(c,c) \notin R(\mathcal{G})$. We have to consider two cases. If $(a,b) \in R(\mathcal{H})$, then $(a,b) \in R(\mathcal{G} \cup \mathcal{H})$. If $(a,b) \in R(\mathcal{F})$, then $(a,b) \in R(\mathcal{G})$ because $\mathcal{F}^{k^*(stg)} = \mathcal{G}^{k^*(stg)}$ is assumed and $(a,b) \in R\left(\mathcal{F}^{k^*(stg)}\right)$ can be derived since neither (a,a) nor (c,c) are included in $R(\mathcal{F})$. In both cases, $(a,b) \in R(\mathcal{G} \cup \mathcal{H})$. Furthermore, $(a,b) \in R\left((\mathcal{G} \cup \mathcal{H})^{k^*(stg)}\right)$ (in contrast to the assumption) since neither (a,a) nor (c,c) are included in $R(\mathcal{G} \cup \mathcal{H})$. □

We have shown that the notion of an stage-*-kernel is robust w.r.t. local expansions. The next technical lemma shows that evaluation given by stage semantics is insensitive w.r.t. transitions to the associated stage-*-kernel.

Lemma 2. *For any AF $\mathcal{F}$, $\mathcal{F} \equiv^{stg} \mathcal{F}^{k^*(stg)}$.*

Proof. At first we show that $\mathcal{F}$ and $\mathcal{F}^{k^*(stg)}$ contain the same conflict-free sets, i.e. $S \in cf(\mathcal{F})$ iff $S \in cf\left(\mathcal{F}^{k^*(stg)}\right)$. The if-direction is given by $R\left(\mathcal{F}^{k^*(stg)}\right) \subseteq R(\mathcal{F})$. It suffices to show that if $S \in cf\left(\mathcal{F}^{k^*(stg)}\right)$, then $S \in cf(\mathcal{F})$. Assume not, i.e. there are two arguments $a, b \in S$, s.t. $(a,b) \in R(\mathcal{F}) \setminus R\left(\mathcal{F}^{k^*(stg)}\right)$. W.l.o.g. $a \neq b$. Consequently, $(a,a) \in R(\mathcal{F})$ or at least $(b,b) \in R(\mathcal{F})$ has to hold. This contradicts the conflict-freeness of S in $\mathcal{F}^{k^*(stg)}$ because $\mathcal{F}^{k^*(stg)}$ and $\mathcal{F}$ share the same self-loops.

We show now that $\mathcal{E}_{stg}(\mathcal{F}) = \mathcal{E}_{stg}\left(\mathcal{F}^{k^*(stg)}\right)$. Let $E \in \mathcal{E}_{stg}(\mathcal{F}) \setminus \mathcal{E}_{stg}\left(\mathcal{F}^{k^*(stg)}\right)$. Hence, $E \in cf(\mathcal{F})$ and thus, $E \in cf\left(\mathcal{F}^{k^*(stg)}\right)$. Furthermore, there exists a conflict-free set E', s.t. $R^+_{\mathcal{F}^{k^*(stg)}}(E) \subset R^+_{\mathcal{F}^{k^*(stg)}}(E')$. We deduce $E' \in cf(\mathcal{F})$ and $E' \not\subseteq E$. Since $R\left(\mathcal{F}^{k^*(stg)}\right) \subseteq R(\mathcal{F})$ and $E \in \mathcal{E}_{stg}(\mathcal{F}) \setminus \mathcal{E}_{stg}\left(\mathcal{F}^{k^*(stg)}\right)$ we deduce the existence of at least two arguments a and b, s.t. $a \in E$, $b \notin E$ and $(a,b) \in R(\mathcal{F}) \setminus R\left(\mathcal{F}^{k^*(stg)}\right)$ because E cannot maintain the same range in $\mathcal{F}^{k^*(stg)}$. Observe that $(a,a) \in R(\mathcal{F})$ yields a contradiction since $E \in cf(\mathcal{F})$ is already deduced. On the other hand, $(c,c) \in R(\mathcal{F})$ for any $c \neq a$ implies that $E = \{a\}$ and $E' \subseteq E$ in contradiction to $E' \not\subseteq E$.

Assume now $E \in \mathcal{E}_{stg}\left(\mathcal{F}^{k^*(stg)}\right) \setminus \mathcal{E}_{stg}(\mathcal{F})$. Thus, E is conflict-free in $\mathcal{F}^{k^*(stg)}$ and $\mathcal{F}$. We deduce the existence of a conflict-free set E', s.t. $R^+_{\mathcal{F}}(E) \subset R^+_{\mathcal{F}}(E')$. Hence, $E' \in cf\left(\mathcal{F}^{k^*(stg)}\right)$ and furthermore, $E \not\subseteq E'$ because $E \in \mathcal{E}_{stg}\left(\mathcal{F}^{k^*(stg)}\right)$ is assumed. Since $R\left(\mathcal{F}^{k^*(stg)}\right) \subseteq R(\mathcal{F})$ and $E \in \mathcal{E}_{stg}\left(\mathcal{F}^{k^*(stg)}\right) \setminus \mathcal{E}_{stg}(\mathcal{F})$ we deduce the existence of at least two arguments a and b, s.t. $a \in E'$, $b \notin E'$ and $(a,b) \in R(\mathcal{F}) \setminus R\left(\mathcal{F}^{k^*(stg)}\right)$ since E' has to reduce its range in $\mathcal{F}^{k^*(stg)}$. Again, $(a,a) \in R(\mathcal{F})$ yields a contradiction because $E' \in cf(\mathcal{F})$ is already shown. Furthermore, if $(c,c) \in R(\mathcal{F})$ for any $c \neq a$ we deduce $E' = \{a\}$ and $E \subseteq E'$ in contrast to $E \not\subseteq E'$. □

We are now prepared to prove the main theorem of this subsection.

Theorem 2. *For any AFs $\mathcal{F}$ and $\mathcal{G}$,*

$$\mathcal{F}^{k^*(stg)} = \mathcal{G}^{k^*(stg)} \Leftrightarrow \mathcal{F} \equiv_L^{stg} \mathcal{G}.$$

Proof. Let $\mathcal{F}^{k^*(stg)} = \mathcal{G}^{k^*(stg)}$ and given an AF $\mathcal{H}$, s.t. $A(\mathcal{H}) \subseteq A(\mathcal{F} \cup \mathcal{G})$. It suffices to show that $E \in \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{H})$ if, and only if $E \in \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{H})$. Suppose $E \in \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{H})$. By Lemma 2, $E \in \mathcal{E}_{stg}\left((\mathcal{F} \cup \mathcal{H})^{k^*(stg)}\right)$ and applying Lemma 1, $E \in \mathcal{E}_{stg}\left((\mathcal{G} \cup \mathcal{H})^{k^*(stg)}\right)$. Finally, using Lemma 2, we derive $E \in \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{H})$ concluding this case. Showing that $E \in \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{H})$ implies $E \in \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{H})$ can be done in a similar way. Consequently, $\mathcal{F} \equiv_L^{stg} \mathcal{G}$ is shown.

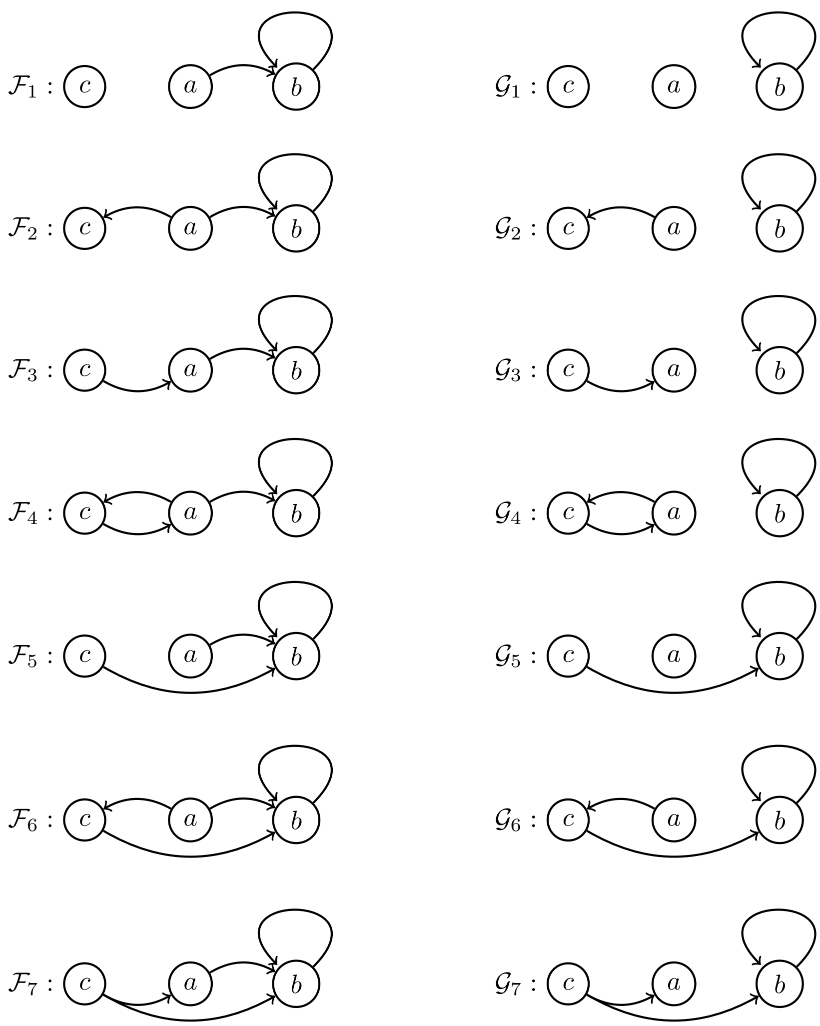
Assume now $\mathcal{F}^{k^*(stg)} \neq \mathcal{G}^{k^*(stg)}$. We will show that $\mathcal{F} \not\equiv_L^{stg} \mathcal{G}$ is implied. W.l.o.g. we may assume $A(\mathcal{F}) = A(\mathcal{G})$ and $(a,a) \in R(\mathcal{F}) \Leftrightarrow (a,a) \in R(\mathcal{G})$ (compare [28, Lemmata 5.3, 5.4]). Consider $a \neq b$ and $(a,b) \in R(\mathcal{F}^{k^*(stg)}) \setminus R(\mathcal{G}^{k^*(stg)})$. It follows that $(a,b) \in R(\mathcal{F})$, $(a,a) \notin R(\mathcal{F})$ and consequently, $(a,a) \notin R(\mathcal{G})$. Now we have to distinguish two cases w.r.t. the presence or absence of the self-loop (b,b).

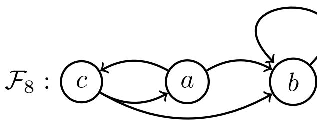
1st case: Assume $(b,b) \notin R(\mathcal{F})$. Thus, $(b,b) \notin R(\mathcal{G})$ and consequently, $(a,b) \notin R(\mathcal{G})$. Note the attack (b,a) may or may not be in $R(\mathcal{F})$ or $R(\mathcal{G})$. We define

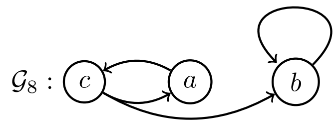
$$\mathcal{K} = (A(\mathcal{F}), \{(a,c), (b,c), (c,c) \mid c \in A(\mathcal{F}) \setminus \{a,b\}\}).$$

In any case, $\{a\} \in \mathcal{E}_{stb}(\mathcal{F} \cup \mathcal{K})$ and consequently, $\{a\} \in \mathcal{E}_{stg}(\mathcal{F} \cup \mathcal{K})$. On the other hand, we state $\{a\} \notin \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{K})$ because $\{\{a,b\}\} = \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{I})$ if $(b,a) \notin R(\mathcal{G})$ and $\{\{b\}\} = \mathcal{E}_{stg}(\mathcal{G} \cup \mathcal{K})$ if $(b,a) \in R(\mathcal{G})$.

2nd case: Consider $(b,b) \in R(\mathcal{F})$. Thus, $(b,b) \in R(\mathcal{G})$ and furthermore, there exists an argument c, s.t. $(c,c) \notin R(\mathcal{F})$. Consequently, $(c,c) \notin R(\mathcal{G})$ and therefore, $(a,b) \notin R(\mathcal{G})$. Unfortunately, there are $2^5 = 32$ possibilities w.r.t. the presence or absence of $(b,a),(b,c),(c,b),(a,c)$ and (c,a) in $R(\mathcal{F})$ and $R(\mathcal{G})$. Consequently, there are $2^{10} = 1024$ combination possibilities of $\mathcal{F}$ and $\mathcal{G}$ w.r.t. to the aforementioned attacks. Fortunately, we do not have to consider every single possibility because we have already shown that if two AFs possess the same stage-*-kernel, then local expansion equivalence of them is implied. Since $(b,b) \in R(\mathcal{F})$ and consequently, $(b,b) \in R(\mathcal{G})$ is assumed we may omit the consideration of (b,a) and (b,c) in $R(\mathcal{F})$ and $R(\mathcal{G})$. Thus, for both AFs $2^3 = 8$ possibilities w.r.t. the presence or absence of $(c,b),(a,c)$ and (c,a) remain. For clarity, we will present all possibilities.







In order to prove that any pair of AFs $(\mathcal{F}_i, \mathcal{G}_j)$ for $1 \leq i, j \leq 8$ can be distinguished by a local expansion we present the following table. An empty cell $(\mathcal{F}_i, \mathcal{G}_j)$ in the table means that $\mathcal{E}_{stg}(\mathcal{F}_i \cup \mathcal{L}) \neq \mathcal{E}_{stg}(\mathcal{G}_j \cup \mathcal{L})$ where

$$\mathcal{L} = (A(\mathcal{F}), \{(a,d), (c,d), (d,d) \mid d \in A(\mathcal{F}) \setminus \{a,b,c\}\}).$$

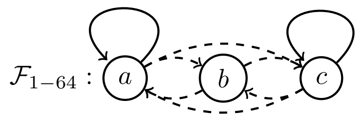
Furthermore, an entry "(a,b)" means that $\mathcal{E}_{stg}(\mathcal{F}_i \cup \mathcal{L}') \neq \mathcal{E}_{stg}(\mathcal{G}_j \cup \mathcal{L}')$ is fulfilled if we consider $\mathcal{L}' = \mathcal{L} \cup (\{a,b\}, \{(a,b)\})$. For instance, $\mathcal{E}_{stg}(\mathcal{F}_8 \cup \mathcal{L}') = \{\{a\},\{c\}\} \neq \{\{a\}\} = \mathcal{E}_{stg}(\mathcal{G}_4 \cup \mathcal{L}')$

	$\mathcal{G}_1$	$\mathcal{G}_2$	$\mathcal{G}_3$	$\mathcal{G}_4$	$\mathcal{G}_5$	$\mathcal{G}_6$	$\mathcal{G}_7$	$\mathcal{G}_8$
$\mathcal{F}_1$	(c,a)				(c,a)			
$\mathcal{F}_2$		(a,c) (c,a)		(a,c) (c,a)		(a,c) (c,a)		
$\mathcal{F}_3$		(a,c) (c,a)		(a,c) (c,a)		(a,c) (c,a)		
$\mathcal{F}_4$		(a,c) (c,a)		(a,c) (c,a)		(a,c) (c,a)		
$\mathcal{F}_5$	(a,c) (c,b)				(a,c) (c,b)			
$\mathcal{F}_6$		(c,b)						
$\mathcal{F}_7$			(a,b)				(a,c)	(a,c)
$\mathcal{F}_8$				(a,b)		(a,b)		

Finally, we have shown that $\mathcal{F} \not\equiv^{stg}_{L} \mathcal{G}$ concluding the proof. □

We want to conclude this section by providing an example showing that equivalence classes w.r.t. local expansion equivalence and stage semantics may be very huge sets in the presence of self-loops.

Example 5. *The following figure represents 64 different AFs (any combination of dashed arrows is suitable). All of them are local expansion equivalent w.r.t. stage semantics.*



3.3 Filling the Gaps: Missing Characterizations for Naive and Eager Semantics

In this section we will prove that normal and strong expansion equivalence w.r.t. naive semantics can be uniformly characterized by the following two conditions: First, possessing the same naive extensions and second, sharing the same arguments. This means, in case of naive semantics expansion, normal expansion, strong expansion and local expansion equivalence coincide. Furthermore, the latter statement holds for eager semantics too but the characterization can be done by the already known admissible-kernel firstly introduced in [34].

Theorem 3. *For any AFs $\mathcal{F}$, $\mathcal{G}$ and $\Phi \in \{N, S\}$,*

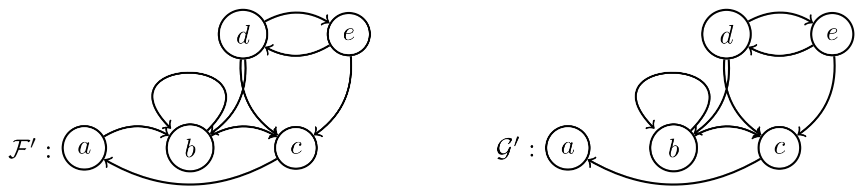
$$\mathcal{E}_{na}(\mathcal{F}) = \mathcal{E}_{na}(\mathcal{G}) \text{ and } A(\mathcal{F}) = A(\mathcal{G}) \Leftrightarrow \mathcal{F} \equiv^{na}_{\Phi} \mathcal{G}.$$

Proof. It suffices to prove that for any $\Phi \in \{N, S\}$, $\mathcal{F} \equiv^{na}_{\Phi} \mathcal{G}$ implies $\mathcal{E}_{na}(\mathcal{F}) = \mathcal{E}_{na}(\mathcal{G})$ and $A(\mathcal{F}) = A(\mathcal{G})$. The converse direction is given by Theorem 5.13 [28] and Proposition 1. First, assuming $\mathcal{E}_{na}(\mathcal{F}) \neq \mathcal{E}_{na}(\mathcal{G})$ immediately entails $\mathcal{F} \not\equiv^{na}_{\Phi} \mathcal{G}$ (Proposition 1). Consider now $A(\mathcal{F}) \neq A(\mathcal{G})$ and $\mathcal{E}_{na}(\mathcal{F}) = \mathcal{E}_{na}(\mathcal{G})$. W.l.o.g. there exists an argument $a \in A(\mathcal{F}) \setminus A(\mathcal{G})$. Obviously, for any $E \in \mathcal{E}_{na}(\mathcal{G})$, $a \notin E$. Consider $\mathcal{H} = (\{a\}, \emptyset)$. We have $a \in E'$, for any $E' \in \mathcal{E}_{na}(\mathcal{G} \cup \mathcal{H})$. Furthermore, $E' \notin \mathcal{E}_{na}(\mathcal{F} \cup \mathcal{H})$ since $\mathcal{F} \cup \mathcal{H} = \mathcal{F}$. Observe that $\mathcal{G} \cup \mathcal{H}$ is a strong and therefore a normal expansion of $\mathcal{G}$ because we added an isolated argument. Consequently, for any $\Phi \in \{N, S\}$, $\mathcal{F} \not\equiv^{na}_{\Phi} \mathcal{G}$. □

Finally, let us consider eager semantics. Remember that eager semantics always returns a unique extension containing the unique ideal extension. We will see that both semantics behave in the same manner w.r.t. the characterization in case of normal expansion equivalence but differ w.r.t. strong expansion equivalence. In fact, in case of strong expansions, eager semantics can be characterized by the well-known admissible-kernel whereas ideal semantics is captured by the more sophisticated admissible-*-kernel firstly introduced in [7]. The following example illustrates this subtle difference.

Example 6. *The attack $(a, b) \in R(\mathcal{F})$ disappears in $\mathcal{F}^{k^*(ad)}$ since we have $\{(b, b), (b, c), (c, a)\} \subseteq R(\mathcal{F})$. Thus, $\mathcal{F}^{k^*(ad)} = \mathcal{G}^{k^*(ad)} = \mathcal{G}$ and consequently, $\mathcal{F}$ and $\mathcal{G}$ are strong expansion equivalent w.r.t. ideal semantics.*

On the other hand, the AFs $\mathcal{F}'$ and $\mathcal{G}'$ show that the initial frameworks $\mathcal{F}$ and $\mathcal{G}$ can be distinguished w.r.t. eager semantics and strong expansions. In fact, $\{a, d\}$ and $\{a, e\}$ are semi-stable extensions in $\mathcal{F}'$ and furthermore, $\{a\}$ is not admissible. Consequently, $\mathcal{E}_{eg}(\mathcal{F}') = \{\emptyset\}$. In case of $\mathcal{G}'$ we observe that $\{a, d\}$ is the unique semi-stable extension. Thus, $\mathcal{E}_{eg}(\mathcal{G}') = \{\{a, d\}\}$.



Theorem 4. *For any AFs $\mathcal{F}$, $\mathcal{G}$ and $\Phi \in \{N, S\}$,*

$$\mathcal{F}^{k(ad)} = \mathcal{G}^{k(ad)} \Leftrightarrow \mathcal{F} \equiv^{eg}_{\Phi} \mathcal{G}.$$

Proof. The if-direction is given by [34, Theorem 2] and Proposition 1. Thus, it suffices to show that $\mathcal{F} \equiv^{eg}_{S} \mathcal{G} \Rightarrow \mathcal{F}^{k(ad)} = \mathcal{G}^{k(ad)}$. We will not present the full proof because in [7, Theorem 5] it is already shown that the admissible kernel adequately determines strong expansion equivalence w.r.t. semi-stable semantics. All parts of this proof may be used to show $\mathcal{F}^{k(ad)} \neq \mathcal{G}^{k(ad)} \Rightarrow \mathcal{F} \not\equiv^{eg}_{S} \mathcal{G}$. The crucial point is that AFs possessing different unique preferred (or semi-stable) extensions serve as an example for AFs with different eager extensions (compare Case 1 - Case 2.2.3).

Consider again Case 2.2.4. We already observed that $\mathcal{E}_{ad}(\mathcal{F} \cup \mathcal{S}) = \mathcal{E}_{ad}(\mathcal{G} \cup \mathcal{S}) = \{\emptyset, \{a, c\}, \{c\}, \{d\}\}$. Furthermore, one may easily check that $\{\{a, c\}\} = \mathcal{E}_{ss}(\mathcal{F} \cup \mathcal{S})$ and $\{\{c\}, \{d\}\} = \mathcal{E}_{ss}(\mathcal{G} \cup \mathcal{S})$. Thus, $\mathcal{E}_{eg}(\mathcal{F} \cup \mathcal{S}) = \{\{a, c\}\} \neq \{\emptyset\} = \mathcal{E}_{eg}(\mathcal{G} \cup \mathcal{S})$. Altogether, $\mathcal{F} \not\equiv^{eg}_{S} \mathcal{G}$ concluding the proof. $\square$

4 Summary of Results and Implications

	stg	stb	ss	eg	ad	pr	id	gr	co	na
L	$k^*(stg)$	$[34]_9$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$[34]_{10}$	$[34]_{11}$	$[28]_{5.13}$
E	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$[28]_{5.13}$
N	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(ad)$	$k(gr)$	$k(co)$	$[28]_{5.13}$
S	$k(stb)$	$k(stb)$	$k(ad)$	$k(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(ad)$	$k^*(gr)$	$k^*(co)$	$[28]_{5.13}$

Figure 3: The whole landscape of characterizations

Figure 3 shows the entire collections of characterizations for the different equivalence notions studied in this paper. Compared to Figure 2 where we have reported the state of the art, we were able to complete the picture by providing:

- a novel kernel for local expansions equivalence in terms of stage semantics, thus rephrasing the result from [27] in terms of a kernel.
- a proof that normal and strong expansion equivalence in terms of stage semantic coincides with expansion equivalence; using the result from [27] that expansion equivalence for stage and stable semantics is the same concept, we thus can use the stable-kernel also to decide normal and strong expansion equivalence in terms of stage semantics.
- a complete picture for eager semantics by suitably adapting the proof for expansion equivalence from [34].
- Finally, we have shown that an analogous situation is given for naive semantics which follows from results in [27].

The fact that different notions of equivalence might or might not coincide is also interesting from a conceptual point of view. To illustrate this let us have a look on normal and strong expansion equivalence. Recall that normal expansions add new arguments and possibly new attacks which involve at least one of the fresh arguments, while strong expansions (a subclass of normal expansions) restrict the possible attacks between the new arguments and the old ones to a single direction. In dynamic settings, both concepts can be justified in the sense that new arguments might be raised but this will not influence the relation between already existing arguments. For strong expansions, only "strong" arguments will be raised, i.e. arguments which cannot be attacked by existing ones. The corresponding equivalence notions now check whether two AFs are "equally robust" to such new arguments, and indeed, normal expansion equivalence always implies strong expansions equivalence but the other direction is only true for some of the semantics (stable, stage, semi-stable and eager). One interpretation is that when two AFs are not normal expansion equivalent, then this can be made explicit by only posing strong arguments (not attacked by existing ones), while for the other semantics this is not the case. For this particular example, it seems that the notion of admissibility which is more "explicit" in the semantics *ad*, *pr*, *id*, *gr*, *co* is responsible for the fact that frameworks might be strong expansion equivalent but not normal expansion equivalent.

Remember that any arbitrary expansion can be split into a normal and local part (compare Definition 2). So one natural conjecture is that normal and local expansion equivalence jointly imply expansion equivalence. Using the results presented in this paper as well as in [34, 7, 27] we can not only verify the addressed conjecture but even give a significantly stronger result. In fact, the main and quite surprisingly relation for the considered semantics can be briefly and concisely stated as: normal expansion equivalence = expansion equivalence. The following proposition captures this statement.

Proposition 2. *For any semantics* $\sigma \in \{stg, stb, ss, eg, ad, pr, id, gr, co, na\}$ *and two AFs* $\mathcal{F}$ *and* $\mathcal{G}$*:*

$$\mathcal{F} \equiv^{\sigma}_{E} \mathcal{G} \text{ iff } \mathcal{F} \equiv^{\sigma}_{N} \mathcal{G}.$$

In our results the notions of a kernel played a crucial role. Indeed, kernels are interesting from several perspectives: First, they allow to decide the corresponding notion of equivalence by a simple check for topological (i.e. syntactical) equality. Moreover, all kernels we have obtained so far can be efficiently constructed from a given argumentation framework. Finally, all notions of kernels have the particular property that they rely on the presence of self-loops. In other words, in case we do not permit AFs with self-loops, each AF coincides with its kernel. Consequently, known and the new results in this paper lead to the following observation:

Proposition 3. *For any self-loop-free AFs* $\mathcal{F}$*,* $\mathcal{G}$*, any* $\Phi \in \{E, N, S\}$*, and any semantics* $\sigma \in \{stg, stb, ss, eg, ad, pr, id, gr, co\}$*:*

$$\mathcal{F} = \mathcal{G} \text{ iff } \mathcal{F} \equiv^{\sigma}_{\Phi} \mathcal{G}.$$

Moreover, for $\sigma \in \{stg, ss, eg, ad, pr, id\}$

$$\mathcal{F} = \mathcal{G} \text{ iff } \mathcal{F} \equiv^{\sigma}_{L} \mathcal{G}.$$

Another particular property of all kernels obtained so far is that the same arguments have to be present and self-looping in either both or none of the compared frameworks. Formally, we have for any AFs $\mathcal{F}$, $\mathcal{G}$, any $\Phi \in \{E, N, S\}$, and any $\sigma \in \{stg, stb, ss, eg, ad, pr, id, gr, co\}$: if $\mathcal{F} \equiv^{\sigma}_{\Phi} \mathcal{G}$, then $A(\mathcal{F}) = A(\mathcal{G})$, and moreover, for each $a \in A(\mathcal{F})$, $(a, a) \in R(\mathcal{F})$ iff $(a, a) \in R(\mathcal{G})$. The same proposition can be given for $\sigma \in \{stg, ss, eg, ad, pr, id\}$ and local expansion equivalence. Also note that therefore the statements of Proposition 3 already hold if at least one of the two compared frameworks is self-loop free.

In contrast to the statements of Proposition 3, we mention that for AFs with self-loops, many different AFs can have the same kernel, and thus are equivalent to each other. For instance, consider expansion equivalence w.r.t. stable semantics which is characterized by the stable-kernel. Furthermore, let $\mathcal{F}$ be an AF possessing n arguments and m self-loops. Then there exist $2^{m(n-1)}-1$ different AFs which are all expansion equivalent to $\mathcal{F}$ w.r.t. stable semantics.

Finally, let us provide some further relations between different equivalence notions. The following results are direct consequences of [34, Theorem 5] and our results as summarized in Figure 3. How the relations between the equivalence notions based on the weaker σ-*-kernel exactly look like is subject of future work.

Proposition 4. *Let $\sigma_1 \in \{stb, stg\}$, $\Phi_1 \in \{E, N, S\}$. Then, $\mathcal{F} \equiv^{\sigma_1}_{\Phi_1} \mathcal{G} \Rightarrow \mathcal{F} \equiv^{\sigma_2}_{\Phi_2} \mathcal{G}$ holds for all AFs $\mathcal{F}$, $\mathcal{G}$, in the following cases: (1) $\sigma_2 \in \{ss, eg\}$, $\Phi_2 \in \{E, N, S, L\}$; (2) $\sigma_2 \in \{ad, pr, id\}$, $\Phi_2 \in \{E, N, L\}$.*

Proposition 5. *The relation $\mathcal{F} \equiv^{co}_{\Phi_1} \mathcal{G} \Leftrightarrow (\mathcal{F} \equiv^{\sigma_2}_{\Phi_2} \mathcal{G}$ and $\mathcal{F} \equiv^{gr}_{\Phi_1} \mathcal{G})$ holds for all AFs $\mathcal{F}$, $\mathcal{G}$, $\Phi_1 \in \{E, N\}$, and either (1) $\sigma_2 \in \{ss, eg\}$, $\Phi_2 \in \{E, N, S, L\}$ or (2) $\sigma_2 \in \{ad, pr, id\}$, $\Phi_2 \in \{E, N, L\}$.*

5 Summary

In this paper, we have given a complete picture of characterizations for four equivalence notions and ten abstract argumentation semantics, thereby providing novel results for eager, stage and naive semantics. Moreover, we have provided an alternative characterization for local equivalence with respect to stage semantics.

Studying equivalence notions between argumentation frameworks has gained increasing interest recently (see also [40, 1]). In fact, due to the inherently non-monotonic nature of argumentation frameworks, strong notions of equivalence give a handle to decide whether two frameworks represent the same knowledge (even if this knowledge is not captured by the actual extensions, but can be made explicit by augmenting the framework under consideration). In this paper, we have discussed the special role of self-loops in such characterizations. In one word, the presence of self-loops seems to be the crucial feature which separates syntactic equivalence from strong notions of equivalence.

For future work, we mention further semantics which have not investigated w.r.t. all the different equivalence notions we dealt with in this paper. One example is cf2 semantics, and the recently introduced variant, stage2 semantics. For both, strong equivalence has been shown to coincide with syntactical equivalence [23, 28] even if self-loops are permitted. It would be interesting to check whether this behaviour carries over to weaker notions of equivalence. A prominent semantics which has not been considered in terms of equivalence checking at all is the resolution-based grounded semantics [3]. Likewise, robust [30] as well as sustainable and tolerant [15] semantics might be interesting to study in this context, since these approaches actually are designed to treat self-attacks different to standard semantics. Also the investigation of further equivalence notions, in particular weak expansion equivalence and the family of minimal change equivalence relations, is on our agenda. We want to mention that a complete picture including the aforementioned equivalence relations has recently been given for stable and preferred semantics (see [10]). In that paper, not only arbitrary AFs are studied, but also results for the special cases where the AFs do not contain self-loops as well as where two AFs share the same arguments are provided. As a final remark concerning future work, we mention the related notions of succinctness [28] and regularity [8], which also deserve further attention.

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