Weighted Propositional Configuration Logics

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◇ **Our goal:**

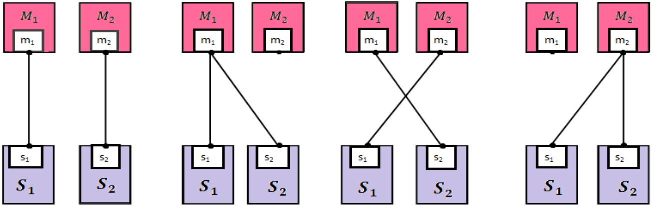
- Describe software architectures with quantitative features.

◇ A new formal method for software architecture description:

- Propositional Configuration Logic.

A. Mavridou, E. Baranov, S. Bliudze, J. Sifakis, Configuration logics: Modelling architecture styles, *J. Log. Algebr. Methods Program.* 86(2016) 2–29. Extended abstract in: *Proceedings of FACS 2015, LNCS* 9539(2015) 256–274.

Software Architecture?



- Components (purple and pink squares).
- Ports on components (white squares).
- Interactions between components (lines).

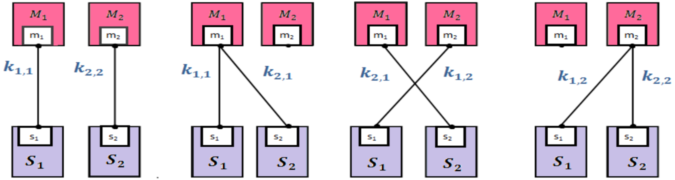
Why do we need them?

- Organise coordination between components.
- Well defined architectures ensure the system to meet its requirements.

Propositionan Configuration Logic (PCL)

- *PCL* describes software architectures.
- Every *PCL* formula can be written in a unique full normal form with which,
  1. the *PCL* is proved to be sound and complete,
  2. and hence the equivalence problem is decidable.
- They constructed a tool using the Maude system, which computes the full normal form for every *PCL* formula and checks the satisfiability for a given set of interactions.

Motivation



Can we compute the

1. minimum cost?
2. maximum probability?
3. energy consumption?

- Weighted configuration logic can describe quantitative features of architectures.
- FO, MSO, LTL with weights have interesting applications (e.g. model checking).

Propositional Interaction Logic (*PIL*)

- **Syntax**

$$\phi ::= true \mid p \mid \overline{\phi} \mid \phi \vee \phi$$

$p \in P$: set of ports.

- **Semantics**
  $I(P) = \mathcal{P}(P) \backslash \{\emptyset\}$: set of all possible interactions between the components.
  $\alpha \in I(P)$, define inductively

$$\begin{array}{ll} \alpha \models_i true, & \\ \alpha \models_i p & \text{if } p \in \alpha, \\ \alpha \models_i \overline{\phi} & \text{if } \alpha \not\models_i \phi, \\ \alpha \models_i \phi_1 \vee \phi_2 & \text{if } \alpha \models_i \phi_1 \text{ or } \alpha \models_i \phi_2. \end{array}$$

Propositional Configuration Logic (*PCL*)

Syntax

$$f ::= true \mid \phi \mid \neg f \mid f \sqcup f \mid f + f$$

ϕ *PIL* formula, $\neg$ complementation operator, $+$ coalescing operator.

- *Closure operator:* $\sim f \stackrel{def}{=} f + true$

Semantics: $\gamma \in C(P) = \mathcal{P}(I(P)) \setminus \{\emptyset\}$,

$\gamma \models true$, always,

$\gamma \models \phi$, if $\forall \alpha \in \gamma,\ \alpha \models_i \phi$ where ϕ is a *PIL* formula,

$\gamma \models \neg f$, if $\gamma \not\models f$,

$\gamma \models f_1 \sqcup f_2$, if $\gamma \models f_1$ or $\gamma \models f_2$,

$\gamma \models f_1 + f_2$, if $\exists \gamma_1, \gamma_2 \in C(P)$, $\gamma_1 \cup \gamma_2 = \gamma$, $\gamma_1 \models f_1$ and $\gamma_2 \models f_2$

Results on *PCL*

$P = \{p_1, \ldots, p_n\}$ set of ports.

- **Monomials**: $p_1 \wedge p_3, \ \overline{p_2} \wedge p_1 \wedge p_4 \wedge \overline{p_8}$
- **Full monomials** (monomials involving all ports):

  e.g. $p_1 \wedge p_2 \wedge \overline{p_3} \wedge \cdots \wedge p_n$

- A *PCL* formula f is in **Full Normal Form** if:

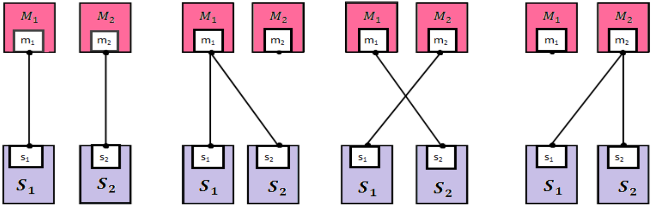
$$f = \bigsqcup_{i \in I} \sum_{j \in J_i} m_{i,j}$$

  where $m_{i,j}$ are **full** monomials.

- Every *PCL* formula f can be written in a unique full normal form.
- *PCL* is sound and complete.

Application on Master/Slave architecture

- $P = \{s_1, s_2, m_1, m_2\}$



$$
\begin{aligned}
f = & (s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2} + s_2 \wedge m_2 \wedge \overline{s_1} \wedge \overline{m_1}) \sqcup \\
& (s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2} + s_2 \wedge m_1 \wedge \overline{s_1} \wedge \overline{m_2}) \sqcup \\
& (s_1 \wedge m_2 \wedge \overline{s_2} \wedge \overline{m_1} + s_2 \wedge m_1 \wedge \overline{s_1} \wedge \overline{m_2}) \sqcup \\
& (s_1 \wedge m_2 \wedge \overline{s_2} \wedge \overline{m_1} + s_2 \wedge m_2 \wedge \overline{s_1} \wedge \overline{m_1})
\end{aligned}
$$

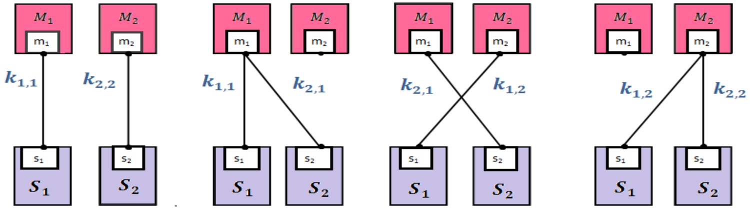
Application on Master/Slave architecture

The full normal form of the formula f for $P = \{m_1, m_2, s_1, s_2\}$ is:

$$\begin{aligned} f = &(s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2} + s_2 \wedge m_2 \wedge \overline{s_1} \wedge \overline{m_1}) \sqcup \\ &(s_1 \wedge m_1 \wedge \overline{s_2} \wedge \overline{m_2} + s_2 \wedge m_1 \wedge \overline{s_1} \wedge \overline{m_2}) \sqcup \\ &(s_1 \wedge m_2 \wedge \overline{s_2} \wedge \overline{m_1} + s_2 \wedge m_1 \wedge \overline{s_1} \wedge \overline{m_2}) \sqcup \\ &(s_1 \wedge m_2 \wedge \overline{s_2} \wedge \overline{m_1} + s_2 \wedge m_2 \wedge \overline{s_1} \wedge \overline{m_1}) \end{aligned}$$

- The unique sets that satisfy the formula are:
  1. $\gamma_1 = \{\{s_1, m_1\}, \{s_2, m_2\}\}$
  2. $\gamma_2 = \{\{s_1, m_1\}, \{s_2, m_1\}\}$
  3. $\gamma_3 = \{\{s_1, m_2\}, \{s_2, m_1\}\}$
  4. $\gamma_4 = \{\{s_1, m_2\}, \{s_2, m_2\}\}$

Weights at interactions



- Consider that each interaction has some kind of "cost". The *weighted PCL*:
  1. characterises the behavior of software architectures considering that every interaction in the architecture has a weight
  2. and computes the minimum/maximum weight that can occur in an architecture by computing the semantics.
- We need an algebraic structure in order to deal with the weights: Commutative semiring.

Weighted *PIL* over *P* and *K*

Syntax

$$\psi ::= k \mid \phi \mid \psi \oplus \psi \mid \psi \otimes \psi$$

$(K, \oplus, \otimes, 0, 1)$ a semiring, $k \in K$, ϕ is a *PIL* formula.

Semantics $\|\psi\| : I(P) \to K$, for $\alpha \in I(P)$:

- $\|k\|(\alpha) = k$,
- $\|\phi\|(\alpha) = \begin{cases} 1 & \text{if } \alpha \models_i \phi \\ 0 & \text{otherwise} \end{cases}$,
- $\|\psi_1 \oplus \psi_2\|(\alpha) = \|\psi_1\|(\alpha) \oplus \|\psi_2\|(\alpha)$,
- $\|\psi_1 \otimes \psi_2\|(\alpha) = \|\psi_1\|(\alpha) \otimes \|\psi_2\|(\alpha)$.

Weighted *PCL* over *P* and *K*

- **Syntax**

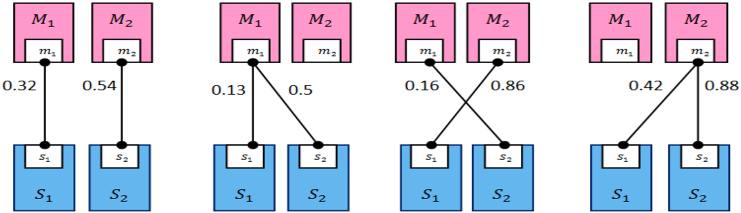
$$\zeta ::= k \mid f \mid \zeta \oplus \zeta \mid \zeta \otimes \zeta \mid \zeta \uplus \zeta$$

$k \in K$, f a *PCL* formula.

Closure operator: $\sim \zeta \overset{def}{=} \zeta \uplus 1$.

- **Semantics** $\|\zeta\| : C(P) \to K$, for $\gamma \in C(P)$,
  - $\|k\|(\gamma) = k$,
  - $\|f\|(\gamma) = \begin{cases} 1 & \text{if } \gamma \models f \\ 0 & \text{otherwise} \end{cases}$,
  - $\|\zeta_1 \oplus \zeta_2\|(\gamma) = \|\zeta_1\|(\gamma) \oplus \|\zeta_2\|(\gamma)$,
  - $\|\zeta_1 \otimes \zeta_2\|(\gamma) = \|\zeta_1\|(\gamma) \otimes \|\zeta_2\|(\gamma)$,
  - $\|\zeta_1 \uplus \zeta_2\|(\gamma) = \bigoplus_{\gamma=\gamma_1 \cup \gamma_2} (\|\zeta_1\|(\gamma_1) \otimes \|\zeta_2\|(\gamma_2))$.

Weighted Master/Slave architecture



- Weighted Interactions formulas for every interaction. For example, for the interaction between the slave S_1 and the master M_1 is:

$$\psi_{1,1} = 0.32 \otimes (s_1 \wedge m_1 \wedge \overline{m_2} \wedge \overline{s_2}) = 0.32 \otimes \phi_{1,1}$$

$\alpha \in I(P)$:

$$\|\psi_{1,1}\|(\alpha) = \|0.32 \otimes \phi_{1,1}\|(\alpha) = 0.32 \otimes \|\phi_{1,1}\|(\alpha)$$

$$= \begin{cases} 0.32 & \text{if } \alpha \models_i \phi_{1,1} \\ 0 & \text{otherwise} \end{cases}$$

Different semiring $\implies$ different result

Weighted *PCL* formula for two Masters and two Slaves:

$$\zeta = \sim ((\psi_{1,1} \oplus \psi_{1,2}) \uplus (\psi_{2,1} \oplus \psi_{2,2}))$$

Let the set $\gamma = \{\{s_1, m_1\}, \{s_1, m_2\}, \{s_2, m_1\}, \{s_2, m_2\}\}$ and,

- min-plus semiring $(\mathbb{R}_+ \cup \{\infty\}, \min, +, \infty, 0)$

$$\begin{aligned}\|\zeta\|(\gamma) &= \min\{0.32 + 0.54, 0.13 + 0.5, 0.16 + 0.86, 0.42 + 0.88\} \\ &= 1.3\end{aligned}$$

returns the minimum "cost",

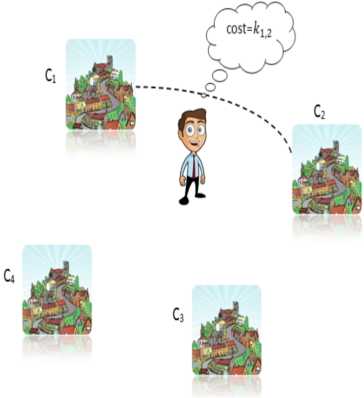
- Viterbi semiring $([0, 1], \max, \cdot, 0, 1)$

$$\begin{aligned}\|\zeta\|(\gamma) &= \max\{0.32 \cdot 0.54, 0.13 \cdot 0.5, 0.16 \cdot 0.86, 0.42 \cdot 0.88\} \\ &= 0.3696\end{aligned}$$

returns the maximum probability.

Travelling Salesman Problem

A different nature problem



Between every two cities there is a "cost". We proved that there exists:

- a weighted *PCL* formula that characterises all the possible routes for a given number of cities with their respectively weights

- and the semantics of the formula considering the $\mathbb{R}_{\min}$ semiring computes the **least cost** that the salesman can achieve.

Results on WPCL

Definition

A weighted *PCL* formula ζ over P and K is in **full normal form** if there are finite index sets I and J_i for every $i \in I$, $k_i \in K$ for every $i \in I$, and full monomials $m_{i,j}$ for every $i \in I$ and $j \in J_i$ such that

$$\zeta = \bigoplus_{i \in I} \left(k_i \otimes \sum_{j \in J_i} m_{i,j} \right).$$

Example

$P = \{p_1, p_2\}$,

$\zeta = (0.5 \otimes (p_1 \wedge p_2 + \overline{p_1} \wedge p_2)) \oplus (2.3 \otimes (p_1 \wedge \overline{p_2} + p_1 \wedge p_2))$

Theorem

Given K, P, ζ,

$$\zeta \xrightarrow[\textit{equivalent}]{\textit{unique}} \zeta'$$

where ζ' is in full normal form.

Theorem

Weighted PCL over P and K is **sound** *and* **complete**.

Theorem

The equivalence problem for weighted PCL formulas is decidable.

Example on equivalence

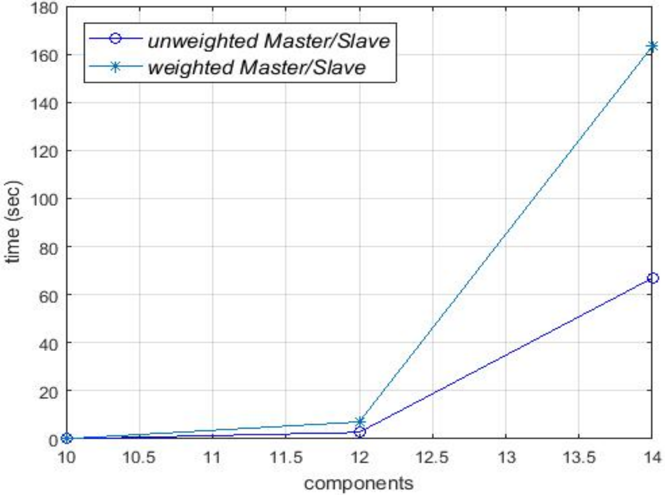
Let $P = \{p_1, p_2, p_3\}$ and the weighted formulas

1. $\zeta_1 = ((0.2 \otimes (p_1 \wedge p_2 \wedge \overline{p_3})) \oplus (0.3 \otimes (p_3 \wedge \overline{p_1} \wedge p_2))) \uplus (1.2 \otimes p_2 \wedge p_1 \wedge p_3)$
2. $\zeta_2 = (1.2 \otimes p_1 \wedge p_2 \wedge p_3 + 0.2 \otimes p_1 \wedge p_2 \wedge \overline{p_3}) \oplus (1.2 \otimes p_1 \wedge p_2 \wedge p_3 + (0.3 \otimes \overline{p_1} \wedge p_2 \wedge p_3))$

$$\zeta_1 \equiv \zeta_2 \ ?$$

- Compute the full normal form for both the weighted formulas.
- They have the same full normal form (which is **unique** for both the weighted formulas ζ_1 and ζ_2), hence they are equivalent.

Full normal form with Maude



Future work

We aim to study:

- whether we can prove our results for the weighted first-order configuration logic (in process),
- weighted second-order configuration logics (in process).

Thank you

$E\upsilon\chi\alpha\rho\iota\sigma\tau\acute{\omega}$

Propositional Interaction Logic

Propositional Configuration Logic

Results on *PCL*

PCL application

Weighted *PIL*

Weighted *PCL*

Application on the weighted Master/Slave

Application on TSP

Results on WPCL

Application with Maude

References

A. Mavridou, E. Baranov, S. Bliudze, J. Sifakis, Configuration logics: Modelling architecture styles, *J. Log. Algebr. Methods Program.* 86(2016) 2–29. Extended abstract in: *Proceedings of FACS 2015, LNCS* 9539(2015) 256–274.

P. Paraponiari, G. Rahonis, On weighted congiguration logics, , in: *Proceedings of FACS 2017, LNCS 10487, pp. 98-116, 2017*, https://arxiv.org/abs/1704.04969