Practical problems with Chomsky-Schützenberger parsing for weighted multiple context-free grammars[1]

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[1]based on T. Denkinger (2017). “Chomsky-Schützenberger parsing for weighted multiple context-free languages”.

The problem: k-best parsing

parsing problem

Input:

- a grammar G
- a word w

Output:

- a derivation of w in G

(not unique)

The problem: k-best parsing

k-best parsing problem [Huang and Chiang 2005]

Input:

- a $(\mathcal{A}, \odot, \mathbb{1}, \mathbb{0})$-weighted grammar (G, wt)
- a *suitable* partial order $\trianglelefteq$ on $(\mathcal{A}, \odot, \mathbb{1}, \mathbb{0})$
- a number $k \in \mathbb{N}$
- a word w

Output:

- a sequence of k best derivations[2] of w in G

(not unique)

[2] w.r.t. *wt* and $\trianglelefteq$ (greater is better)

Multiple context-free grammars

context-free grammars

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multiple context-free grammars

[Seki, Matsumura, Fujii, and Kasami 1991]

$$A \to [\underbrace{((x_1, x_2), (y_1, y_2)) \mapsto (\mathrm{a}x_1 y_2 \mathrm{b}, y_1 \mathrm{c} x_2)}_{(\Sigma^* \times \Sigma^*) \times (\Sigma^* \times \Sigma^*) \to (\Sigma^* \times \Sigma^*)}](A, B)$$

composes *tuples* of strings

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composes *tuples* of strings

$\Longrightarrow$ extra expressive power useful for natural language processing

The Chomsky-Schützenberger theorem

CS-theorem$_S$ [Chomsky and Schützenberger 1963]

Let L be a language. T.f.a.e.

1. $\exists$ CFG G s.t. $L = \mathrm{L}(G)$
2. $\exists$ regular language R,
   $\exists$ Dyck language D,
   $\exists$ homomorphism h
   s.t. $L = h(R \cap D)$

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CS-theorems [Chomsky and Schützenberger 1963] [Yoshinaka, Kaji, and Seki 2010]

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Idea [Hulden 2011, for CFGs]

Use the decomposition provided by (1. $\to$ 2.) for parsing.

From the CS-theorem to CS-parsing

$w \in \mathrm{L}(G)$

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$$\begin{aligned} w \in \mathrm{L}(G) &\iff w \in h(R \cap D) && \text{(CS-theorem)} \\ &\iff \exists u \in R \cap D\colon h(u) = w \\ &\iff \exists u \in R \cap h^{-1}(w)\colon u \in D \end{aligned}$$

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Observation

Each $u \in R \cap D$ encodes a derivation of G.

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k-best CS-parsing

$$\mathrm{parse}_{G,wt,k}(w)$$

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Derivations and bracket words

an MCFG G:

$$\alpha\colon S \to [x_1 x_2](A)$$
$$\beta\colon A \to [\mathrm{a}x_1\mathrm{b}, \mathrm{c}x_2](A)$$
$$\gamma\colon A \to [\varepsilon, \varepsilon]()$$

Derivations and bracket words

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$$\mathrm{L}(G) = \{\mathrm{a}^n \mathrm{b}^n \mathrm{c}^n \mid n \in \mathbb{N}\}$$

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$$S_1 \bullet \qquad \bullet \bar{S}_1$$

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word $w =$

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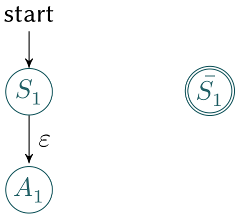

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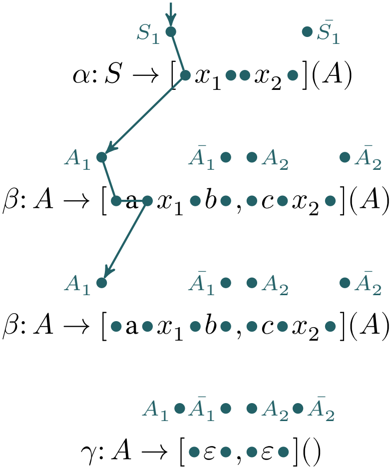
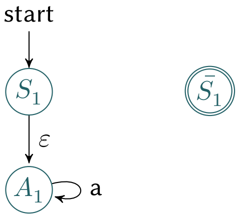


word $w = \mathrm{a}$



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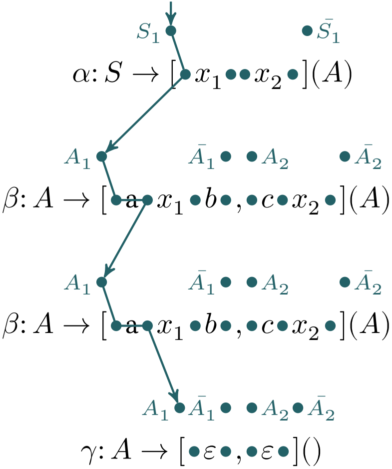
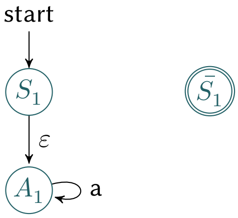
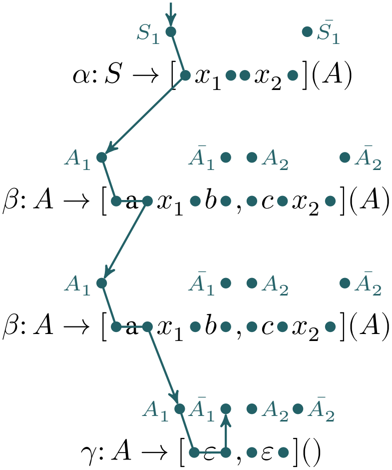


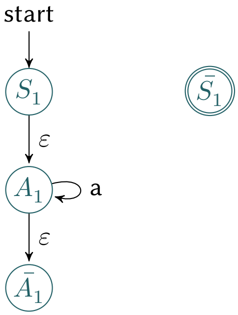


word $w = \mathrm{aa}$

Derivations and bracket words



word $w =$ aa



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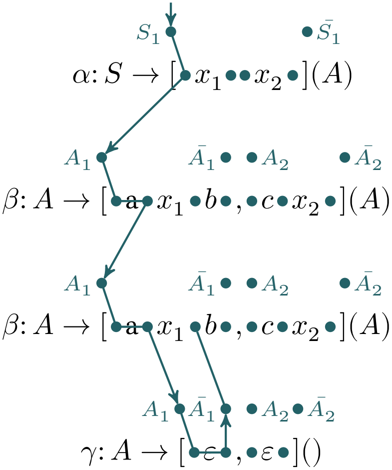
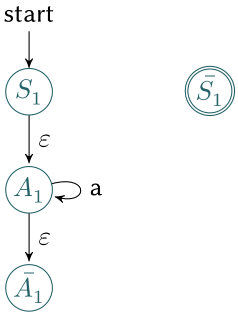


word $w = \text{aa}$



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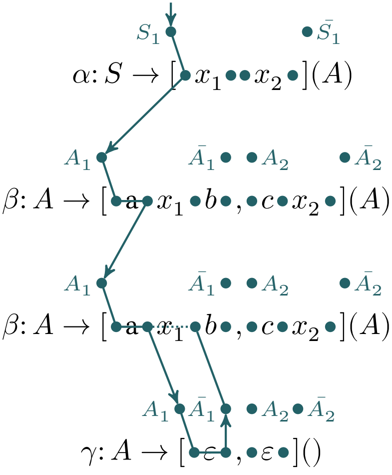
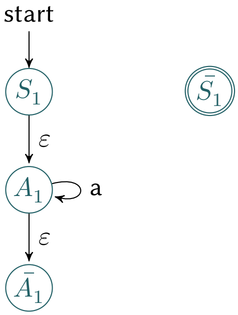


word $w = \text{aa}$



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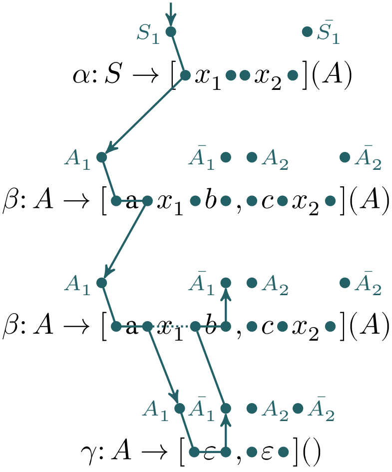
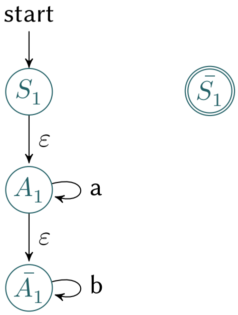


word $w = \text{aab}$



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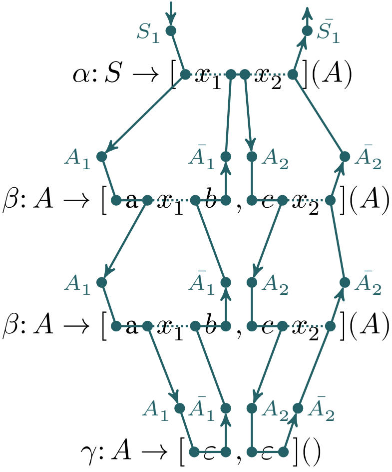
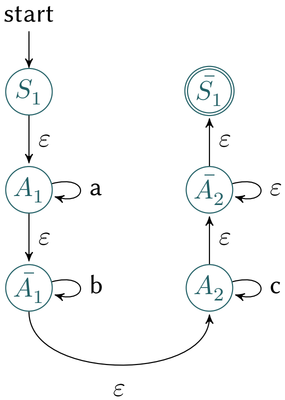


word $w =$ aabbcc



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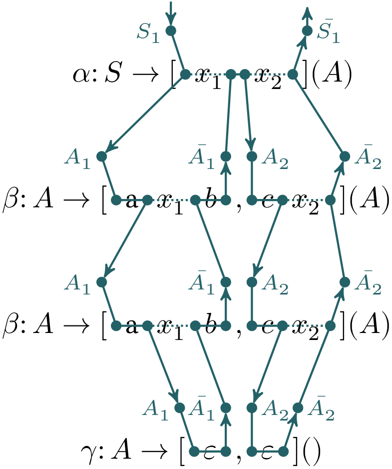
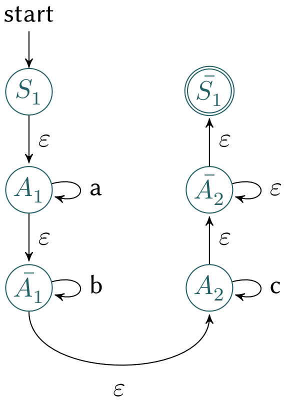
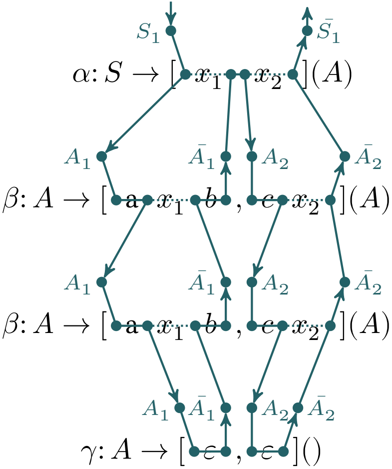


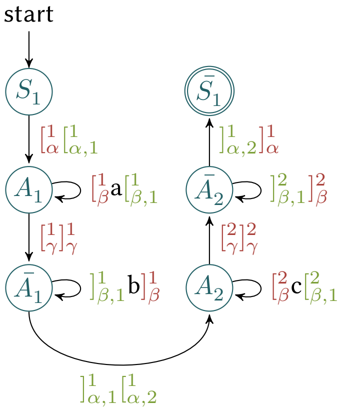


word $w = \text{aabbcc}$

$w' = \text{aaabbcc} \notin \mathrm{L}(G)$

Derivations and bracket words





word $w = \mathrm{aabbcc}$

$w' = \mathrm{aaabbcc} \notin \mathrm{L}(G)$

word $u = [^1_\alpha [^1_{\alpha,1}\ [^1_\beta \mathrm{a} [^1_{\beta,1}\ [^1_\beta \mathrm{a} [^1_{\beta,1}\ [^1_\gamma]^1_\gamma\ ]^1_{\beta,1} \mathrm{b}]^1_\beta\ ]^1_{\beta,1} \mathrm{b}]^1_\beta\ ]^1_{\alpha,1}$
$[^1_{\alpha,2}\ [^2_\beta \mathrm{c} [^2_{\beta,1}\ [^2_\beta \mathrm{c} [^2_{\beta,1}\ [^2_\gamma]^2_\gamma\ ]^2_{\beta,1}]^2_\beta\ ]^2_{\beta,1}]^2_\beta\ ]^1_{\alpha,2}]^1_\alpha$

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$$\begin{aligned} w \in \mathrm{L}(G) &\iff w \in h(R \cap D) \\ &\iff \exists u \in R \cap D\colon h(u) = w \\ &\iff \exists u \in R \cap h^{-1}(w)\colon u \in D \end{aligned}$$

(CS-theorem)

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Each $u \in R \cap D$ encodes a derivation of G.

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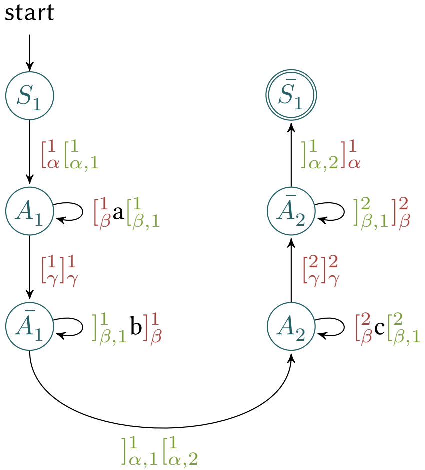
Practical problems

... with the weighted finite state automaton R^{wt}

Practical problems

... with the weighted finite state automaton R^{wt}



Practical problems

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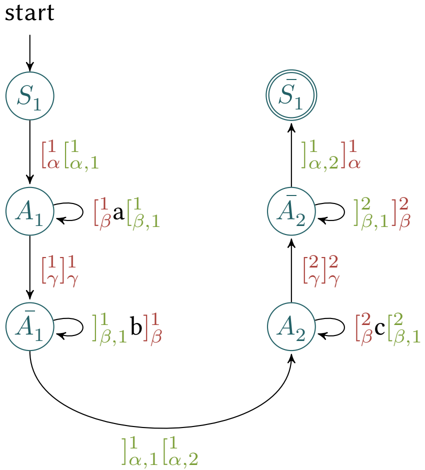

enumerate R^{wt}

- by ascending weight
- *Dijkstra*-like algorithm

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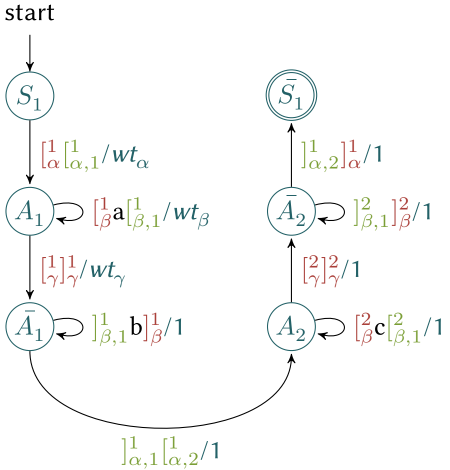


enumerate R^{wt}

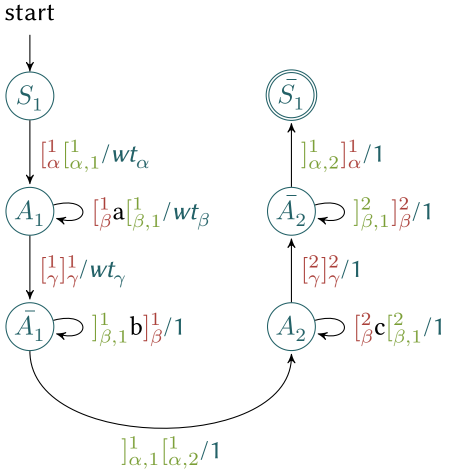
- by ascending weight
- *Dijkstra*-like algorithm

initial idea:

- attach weights to $[^1_\sigma$-brackets

Practical problems

… with the weighted finite state automaton R^{wt}



enumerate R^{wt}

- by ascending weight
- *Dijkstra*-like algorithm

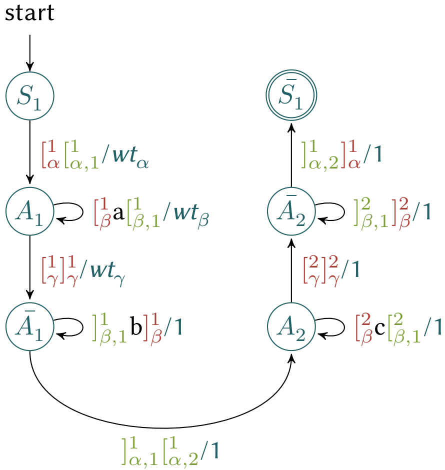
initial idea:

- attach weights to $[^1_\sigma$-brackets

problem:

- loops with weight $\mathbb{1}$

Solutions and workarounds I



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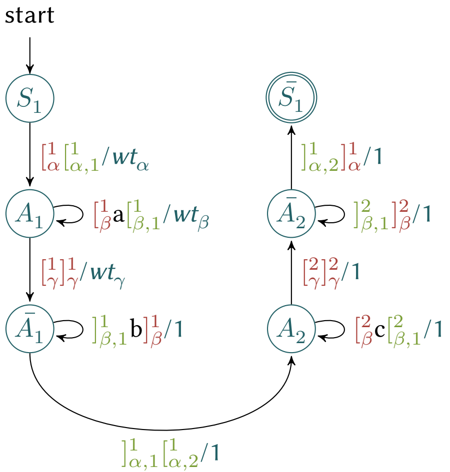

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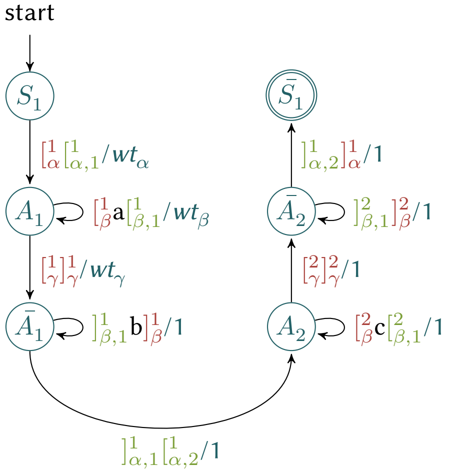

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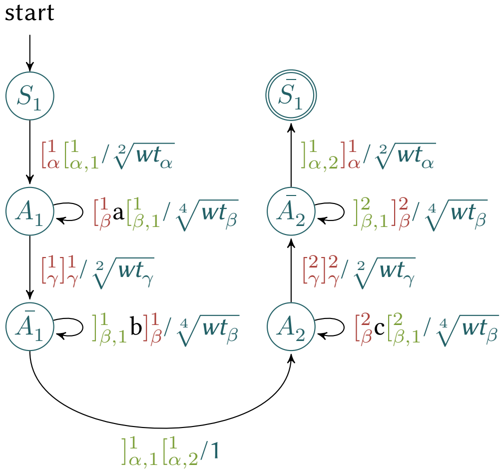

- **assume** that $wt_\sigma \neq \mathbb{1}$ in loops
- **assume** that weights can be *factorised*
- distribute factors of wt_σ among transitions with $[^1_\sigma,]^1_\sigma, [^2_\sigma,]^2_\sigma, \ldots$

Solutions and workarounds II

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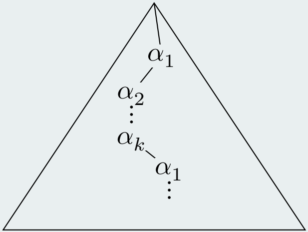

with $wt_{\alpha_1} = \ldots = wt_{\alpha_k} = \mathbb{1}$

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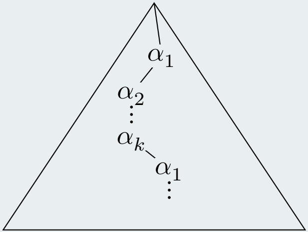

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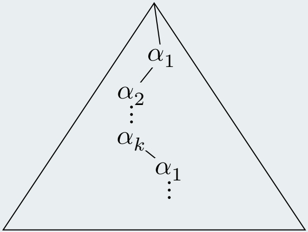

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- *useful* probabilistic MCFGs are always restricted
- $\mathbb{B}$-weighted MCFGs can be transformed to restricted $\mathbb{N}$-weighted MCFGs with the same support

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factorisable (multiplicative) monoid with zero $(\mathcal{A}, \odot, \mathbb{1}, \mathbb{0})$

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two examples from nlp:

$(\mathcal{A}, \odot, \mathbb{1}, \mathbb{0})$	*factorisation*
$([0,1], \cdot, 1, 0)$	$a = \sqrt[2]{a} \cdot \sqrt[2]{a}$
$(\mathbb{R}_{\geq 0}^{-\infty}, +, 0, -\infty)$	$a = a/2 + a/2$

Conclusion and outlook

Theorem (k-best parsing)

Let (G, wt) be a *restricted* weighted MCFG over a *factorisable* monoid with zero and $\trianglelefteq$ be a *suitable* partial order on the monoid. Then

$$(\text{toDeriv} \circ \text{take}_k \circ \text{filter}_{\cap D} \circ \text{sort}_{\trianglelefteq})(R^{wt} \triangleright h^{-1}(w))$$

solves the k-best parsing problem for (G, wt) and a word w.

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Thank you for your attention.

References

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