Weighted Automata – Theory and Applications

Leipzig, March 27 – 31, 2006

edited by Manfred Droste and Heiko Vogler

Preface

This report contains the program and the abstracts of lectures delivered at the workshop "Weighted Automata – Theory and Applications" which took place at Leipzig University, March 27 – 31, 2006. This workshop covered all aspects of weighted automata, ranging from the theory of formal power series to quantum automata and applications e.g. in digital image processing or model-checking of probabilistic systems. The workshop was attended by 45 participants from 12 countries.

Two tutorials were given by

J. Albert (Würzburg),
J. Gruska (Brno).

In addition, eight invited lectures were presented by

C. Baier (Bonn),
S. Bozapalidis (Thessaloniki),
Z. Ésik (Szeged),
P. Gastin (Paris-Cachan),
J. Kari (Turku/Iowa),
W. Kuich (Wien),
I. Meinecke (Leipzig),
G. Rahonis (Thessaloniki).

Furthermore, nine talks were selected as contributed communications.

The workshop was organized jointly by the *Chair for Automata and Formal Languages* of Leipzig University and the *Chair for Foundations of Programming* of Dresden University of Technology. For further financial support we would like to thank the activity "Automata: from Mathematics to Applications" of the *European Science Foundation* and the Graduiertenkolleg "Knowledge Representation" of the *German Research Foundation (DFG)* at Leipzig University.

Call for Papers

The Journal of Automata, Languages, and Combinatorics (JALC) has agreed to publish a special issue on this topic. Submissions related to this topic could be either survey articles or research papers and will be refereed as usual. Participation in the above workshop is encouraged, but is not a prerequisite for a submission.

Authors are asked to submit their contribution preferably in PostScript (or pdf) to both of the editors of the special issue. Please send your files to

`droste@informatik.uni-leipzig.de` and `vogler@inf.tu-dresden.de`.

Deadline for submissions is **May 10, 2006**. We intend to ensure a quick refereeing process. Authors of published papers will be provided with 20 reprints free of charge.

Manfred Droste and Heiko Vogler

Part I
Program

Monday, March 27, 2006

08.30 – 09.00	REGISTRATION	
09.00 – 10.30	J. Albert (Würzburg) *Generating images and videoclips by weighted finite automata*	TUTORIAL
10.30 – 11.00	BREAK	
11.00 – 12.00	W. Kuich (Wien) *Fuzzy regular languages over finite and infinite words*	INVITED LECTURE
12.00 – 14.00	LUNCH	
14.00 – 15.00	J. Kari (Turku/Iowa) *Survey: Finite state methods for image analysis and manipulation*	INVITED LECTURE
15.00 – 15.30	P. Steinby (Turku) *Observations on the smoothness properties of real functions computed by weighted finite automata*	
15.30 – 16.00	BREAK	
16.00 – 16.30	G. Tischler (Würzburg) *Parametric weighted finite automata with unary alphabet*	
16.30 – 17.00	A. Maletti (Dresden) *Preservation of recognizability for* o-*substitution*	

Tuesday, March 28, 2006

08.30 – 09.00	Registration	
09.00 – 10.30	J. Albert (Würzburg) *Generating images and videoclips by weighted finite automata*	Tutorial
10.30 – 11.00	Break	
11.00 – 12.00	P. Gastin (Paris-Cachan) *Weighted automata and weighted logics*	Invited Lecture
12.00 – 14.00	Lunch	
14.00 – 15.00	I. Meinecke (Leipzig) *Weighted concurrency*	Invited Lecture
15.00 – 15.30	I. Mäurer (Leipzig) *Characterizations of recognizable picture series*	
15.30 – 16.00	Break	
16.00 – 16.30	Ch. Mathissen (Leipzig) *Recognizable skew formal power series over trace monoids*	
16.30 – 17.00	S. Schwarz (Halle-Wittenberg) *Łukasiewicz logic and automata over MV-algebras*	
17.00 – 17.30	Short communication & research session	

Wednesday, March 29, 2006

08.30 – 09.00	REGISTRATION	
09.00 – 10.30	J. Gruska (Brno) *Quantum (finite) automata: an invitation*	TUTORIAL
10.30 – 11.00	BREAK	
11.00 – 12.00	W. Kuich (Wien) *Transducers for ω-languages – an algebraic generalization*	INVITED LECTURE
12.00 – 14.00	LUNCH	
14.00 –	EXCURSION & FREE TIME	

Thursday, March 30, 2006

08.30 – 09.00	REGISTRATION	
09.00 – 10.30	J. Gruska (Brno) *Quantum (finite) automata: an invitation*	TUTORIAL
10.30 – 11.00	BREAK	
11.00 – 12.00	C. Baier (Bonn) *Probabilistic ω-automata*	INVITED LECTURE
12.00 – 14.00	LUNCH	
14.00 – 15.00	S. Bozapalidis (Thessaloniki) *Picture deformation*	INVITED LECTURE
15.00 – 15.30	J. Waldmann (Leipzig) *Weighted automata for proving termination of string rewriting*	
15.30 – 16.00	BREAK	
16.00 – 17.30	SHORT COMMUNICATION & RESEARCH SESSION	
19.00 –	CONFERENCE DINNER	

Friday, March 31, 2006

08.30 – 09.00	Registration
09.00 – 10.00	G. Rahonis (Thessaloniki) Invited Lecture *Weighted logics on infinite words and trees*
10.00 – 10.30	Break
10.30 – 11.00	G. Tischler (Würzburg) *Hybridization of near random to random access video compression with weighted finite automata*
11.00 – 11.30	D. Kirsten (Dresden) *A Burnside approach to the termination of Mohri's algorithm for polynomially ambiguous min-plus-automata*
11.30 – 12.30	Lunch

Part II
Abstracts

Tutorials

Generating Images and Videoclips by Weighted Finite Automata

Jürgen Albert

Department of Computer Science, University of Würzburg
Am Hubland, D-97074 Würzburg, Germany
albert@informatik.uni-wuerzburg.de

In a straightforward manner a language $L \subseteq \Sigma^*$ defined by any finite state acceptor A, $L = L(A)$, can be associated with a set of black-and-white images (bi-level images) if the words in Σ^* are interpreted as addresses of pixels. Each intersection $L \cap \Sigma^r$ for some fixed r defines an image of some given resolution.

Let $\Sigma = \{0, 1, \ldots, k-1\}$ contain the digits for the k-ary number representations, then $w = a_1\ a_2\ \ldots a_r$ with $a_i \in \Sigma$ addresses the half-open interval $[0.a_1a_2 \ldots a_r,\ 0.a_1a_2 \ldots a_r + k^{-r}) \subseteq [0, 1)$ of length k^{-r} and this interval is assigned the value 1 (has the color white) if $w \in L$, else it has the value 0 and is painted black. Here it is only a matter of interpretation, whether parts of the unit interval, the unit square or unit cube are considered. E.g. it is very common to address subquadrants in the unit square via the alphabet $\Sigma = \{0, 1, 2, 3\}$. There is a natural correspondence with quadtrees (bintrees and octrees for the dimensions 1 or 3 resp.) and the unrolling of the finite automaton for all input strings of a certain fixed length. Likewise, different recursive traversals exist for the sets of pixels, most notably the so-called Morton- or Z-order, which works like zooming into the next level of detail. Among the many variations of the Z-order we also find the Hilbert-Peano traversal, a well-known space filling curve.

In the early nineties Weighted Finite Automata, WFA for short, were introduced and studied by K. Culik II, J. Karhumäki, J. Kari et al. as finite state generators for smooth and fractal-like real-valued functions over the unit interval or unit square. WFA extend ordinary nondeterministic finite automata by labeling all states and transitions with real numbers. Consuming the next input symbol and following the available transitions in the automaton now means evaluating a linear combination. WFA can provide very efficient representations for polynomials and many more interesting functions like e.g. specific wavelets. The real values assigned to the pixel-addresses form a grey-scale image for the given resolution and given any grey-scale image there trivially exists a (possibly huge) WFA representing this image. The steps from bi-level images to cartoons with only a few colors or textures and from grey-scale to "true-color" images with more than 16 million different colors are not hard since each color layer (e.g. red, green, blue) can be treated separately by a WFA.

In 1993 K. Culik II and J. Kari showed that a cost-driven recursive inference algorithm effectively exploits self-similarities in images and produces small

WFA for high quality approximations of given grey-scale or color images. The compression performance is in general superior to the well-known JPEG Baseline algorithm, especially for high compression rates (”low bitrate-coding”). For cartoons the inference algorithm is adapted in that linear combinations in WFA can be replaced by Boolean expressions. Since WFA are very general concepts, they are well suited for all kind of hybrid approaches and extensions. These include wavelet-transformations, control of reconstruction quality for ”regions of interest” as found in e.g. JPEG2000 as well as handling sequences of pictures as found in video-clips, 3D medical imaging or animations. The building blocks from established MPEG-standards like ”group of pictures” (GOPs) or ”motion vectors” can cooperate here very well with the WFA-representations.

INVITATION to QUANTUM (FINITE) AUTOMATA

Jozef Gruska
Faculty of Informatics, Masaryk University
Botanická 68a, 60200 Brno, Czech Republik

ABSTRACT

Quantum automata are, on one side, models of automata that go, in a natural way, beyond models of probabilistic automata and, on the other side, a natural way to get a deeper inside into the power of different quantum information processing features and phenomena. One can also say that, on one side, study of quantum automata is a natural way how to utilize further and sharpen techniques that have been successful at the study of the classical automata and, on the other side, a way of developing of techniques that can be useful not only in discovering potential of quantum information processing, but also for study of classical models of automata.

In the first half of the tutorial, basics of quantum information processing and communication will be presented for pedestrians: why is QIPC interesting and important, what are specific resources and main models of QIPC and, finally, several very fundamental, but easy to demonstrate, results and limitations of QIPC will be introduced: no-cloning theorem, teleportation, non-locality boxes, Bell inequalities, quantum one-time pad, "classical" and unusual universal sets of quantum computational primitives. The main goal of this part of the tutorial is to show beauty, mystery, importance, main models and concepts as well as simplicity of the basic laws and limitations of QIPC.

In the second half of the tutorial, at first several ways and main problems of "quantumization" of classical models of automata will be discussed. After that main models, computation modes, examples and results concerning power and succinctness of quantum finite automata will be introduced, discussed and analysed in some details. The main goal of this part of the tutorial is to show how to quantumize classical automata and what kind of old and new problems are of importance and interest at the study of such models.

Invited Lectures

Probabilistic ω-automata

by Christel Baier, Universität Bonn, Germany

Probabilistic finite automata as acceptors for languages over finite words have been studied by many researchers. In this talk, we discuss probabilistic automata for recognizing languages over infinite words. The idea is to resolve the choices in a nondeterministic ω-automaton by probabilities and to require positive probabilities for the accepting runs. Surprisingly, probabilistic Büchi automata are more expressive than non-deterministic ω-automata. However, a certain subclass of probabilistic Büchi automata can be identified that has exactly the power of ω-regular languages. Concerning the efficiency, probabilistic ω-automata are not comparable with their nondeterministic counterparts. There are examples of ω-regular languages that have probabilistic Büchi automata of polynomial size, while any nondeterministic ω-automata with Streett, Rabin or Büchi acceptance is of exponential size.
The talk will introduce the formal notion of probabilistic automata with Büchi and other acceptance conditions, discuss their expressiveness and efficiency and some basic properties.

Picture Deformation

Symeon BOZAPALIDIS
Department of Mathematics
Aristotle University of Thessaloniki
54006 Thessaloniki, Greece
e-mail: bozapali@math.auth.gr

In algebraic picture processing, next basic problems arise:

1. Picture Grammar generation demands rules p → q with p, q pictures with the same rank which leads to trivial language generation.

2. The most used picture morphism is projection which sends pixels to pixels which is a very restricted class of picture transformations.

3. Traditionally, infinite pictures are constructed in an external way and this augments their rank up to infinity.

On the other hand, one cannot speak of infinite pictures of finite rank. All these phenomena are exclusively due to the non flexibility of the used pixels.

In order to overcome the above difficulty we here propose the notion of picture deformation.

Thus to any pixel x a family of pixels $x^{(r,s)}$ is attached, called the *(r, s) - deformed pixels* of x, where r, s range over a semiring A.

Hence to any picture constructed by pixels in an alphabet X, its (r, s) - deformation results by replacing all pixels occuring in p by their corresponding (r, s) - deformations.

It turns out that when we consider picture grammars, we actually use a finite *scheme* of *rules* which means that together with a rule $x \to p$ we may use all its (r, s) - deformations.

Clearly, during the derivation the clarity of the generated picture increases, hence the denomination picture clarity grammars for the so used systems.

Picture substition follows an analogous machinery principle: if we want to substitute the pictures $p_1, ..., p_k$ at the places of deformed variable pixels $x_1, ..., x_k$ occuring in a picture p, we only have to substitute the appropriate deformations of $p_1, ..., p_k$ at the corresponding variables. Picture languages generated by clarity grammars as described previously are also obtained as components of the least solution of a picture system with the above mentioned substitution operation and vice versa.

Iteration theories and weighted automata

Zoltan Ésik
University of Szeged

I will review the concept of iteration theories and present some applications to weighted automata.

Weighted automata and weighted logics*

Manfred Droste[1] and Paul Gastin[2]

[1] Institut für Informatik, Universität Leipzig
Augustusplatz 10-11, D-04109 Leipzig, Germany,
droste@informatik.uni-leipzig.de
[2] LSV, ENS de Cachan & CNRS
61, Av. du Président Wilson, F-94235 Cachan Cedex, France,
Paul.Gastin@lsv.ens-cachan.fr

Weighted automata are used to describe quantitative properties in various areas such as probabilistic systems, image compression, speech-to- text processing. The behaviour of such an automaton is a mapping, called a formal power series, assigning to each word a weight in some semiring. We generalize Büchi's and Elgot's fundamental theorems to this quantitative setting. We introduce a weighted version of MSO logic and prove that, for commutative semirings, the behaviours of weighted automata are precisely the formal power series definable with our weighted logic. We also consider weighted first-order logic and show that aperiodic series coincide with the first-order definable ones, if the semiring is locally finite, commutative and has some aperiodicity property. An extended abstract of this work was published in [DG05].

References

[DG05] M. Droste and P. Gastin. Weighted automata and weighted logics. In *Automata, Languages and Programming (32nd ICALP, Lissabon)*, volume 3580 of *Lecture Notes in Comp. Sc.*, pages 513–525. Springer, 2005.

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Survey:

Finite state methods for image analysis and manipulation

Jarkko Kari
Department of Mathematics
FIN-20014 University of Turku, Finland
email: `jkari@utu.fi`

Weighted finite automata (WFA) can be used as compact representations of images. In this survey we review basic algorithms related to finding a WFA representation of a given image and we discuss the state minimization process. We also define weighted finite transducers (WFT) and show how they act on images. We show several interesting examples of image transformations that can be implemented using WFT's. We also demonstrate how WFT transformations can be done directly on the WFA that represents the image, without decoding the image first.

Fuzzy regular languages over finite and infinite words

Werner Kuich
Institut für Algebra und Computermathematik
Technische Universität Wien
Wiedner Hauptstrae 8-10, A-1040 Wien, Austria
kuich@tuwien.ac.at

George Rahonis
Department of Mathematics
Aristotle University of Thessaloniki
54124 Thessaloniki, Greece
grahonis@math.auth.gr

ABSTRACT

We consider finite automata over semirings and quemirings accepting finite and infinite words, respectively. We obtain Kleene theorems for fuzzy languages consisting of finite and infinite words. Furthermore, we introduce regular fuzzy grammars and linear fuzzy systems and we show that both of them specify the class of recognizable fuzzy languages consisting of finite and infinite words.

To appear in: *Fuzzy Sets and Systems.*

Transducers for ω-languages—an algebraic generalization

Zoltán Ésik*
University of Szeged
ze@inf.u-szeged.hu

Werner Kuich*
Technische Universität Wien
kuich@tuwien.ac.at

We introduce rational and algebraic transducers, and abstract families of elements over starsemiring-omegasemimodule pairs and prove that rational and algebraic closures are such abstract families of elements. Furthermore, we specialize our results to rational and algebraic transducers in the classical sense and to abstract families of ω-languages.

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Weighted concurrency

Ingmar Meinecke

Institut für Informatik, Universität Leipzig,
Augustusplatz 10–11, D-04109 Leipzig, Germany
meinecke@informatik.uni-leipzig.de

We give a survey about some weighted models of concurrency, show several characterizations of these models, and discuss the benefit of quantitative aspects for concurrency.

Mazurkiewicz traces model concurrency by a global independence relation on an alphabet. We consider trace series, i.e., functions from a trace monoid $\mathbb{M}$ into a semiring $\mathbb{K}$. Recognizable trace series are defined by a finite linear representation. For a commutative semiring $\mathbb{K}$, the classes of recognizable and mc-rational trace series coincide [DG99]. Moreover, recognizable trace series can be characterized by a restricted weighted MSO-logic [Mei06]. Asynchronous automata are an important recognizing device for traces. Recently, the equivalence of recognizable trace series with those recognized by a weighted asynchronous cellular automaton was shown [Kus06].

Moreover, for traces we get an equivalence of aperiodic, star-free, and FO-definable series under certain restrictions on the semiring $\mathbb{K}$ [DG00,Mei06].

Furthermore, we will investigate how to treat quantitative aspects of sequentiality and concurrency differently. In weighted automata over words, sequential composition is modeled by the multiplication of the underlying semiring, and non-determinism by the addition of the semiring. If we consider an additional parallel composition, we should also introduce a second multiplication to deal with parallelism in the weight structure. This concept is realized by series-parallel posets and functions from sp-posets into a bisemiring, equipped with a sequential and a parallel multiplication. Here, the recognizing device is a weighted branching automaton with weights from a bisemiring. We present a result in the spirit of Schützenberger for sp-series [KM04] and indicate further closure properties of recognizable sp-series [Mei05].

Last but not least, we discuss first results for more general weighted asynchronous cellular automata over directed acyclic graphs, and their possible consequences.

References

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[KM04] D. Kuske and I. Meinecke. Branching automata with costs – a way of reflecting parallelism in costs. *Theoret. Comp. Sc.*, 328:53–75, 2004.

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Weighted logics on infinite words and trees

George Rahonis
Department of Mathematics
Aristotle University of Thessaloniki
54124 Thessaloniki, Greece
grahonis@math.auth.gr

This lecture consists of two parts. The first part is a presentation of a recent work of Droste and Rahonis [1]. More precisely, we introduce weighted Muller automata over infinite words and we establish their basic properties. Then we define a weighted MSO logic and we interpret the semantics of weighted MSO formulas as formal power series over infinite words. We show that the class of the behaviors of weighted Muller automata coincides with the semantics of restricted weighted MSO formulas, provided that the underlying semiring is totally commutative complete. Finally, we prove that our automata are equivalent to weighted Büchi automata of Ésik and Kuich [3].

In the second part, we define weighted Muller tree automata and we state the basic properties of their behaviors. Furthermore, we introduce a weighted MSO logic on infinite trees and we investigate the relation among the semantics of weighted MSO formulas and the behaviors of weighted Muller tree automata over certain semirings. This work is an extension of the work of Droste and Vogler [2].

References

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[2] M. Droste, H. Vogler, Weighted Tree Automata and Weighted Logics (submitted).

[3] Z. Ésik, W. Kuich, A semiring-semimodule generalization of ω-regular languages I, II. Special issue on "Weighted automata" (M. Droste, H. Vogler, eds.) *J. of Automata Languages and Combinatorics,* to appear.

Technical Contributions

A Burnside Approach to the Termination of Mohri's Algorithm for Polynomially Ambiguous Min-Plus-Automata

Daniel Kirsten

Dresden University of Technology, Institute of Algebra, 01062 Dresden, Germany
http://www.math.tu-dresden.de/∼kirsten/

We show that the termination of MOHRI's algorithm is decidable for polynomially ambiguous weighted finite automata over the tropical semiring which gives a partial answer to a question by MOHRI from 1997 [2]. The proof relies on an improvement of the notion of the twins property and a Burnside type characterization for the finiteness of the set of states produced by MOHRI's algorithm. This Burnside type characterization states that MOHRI's algorithm terminates on trim, polynomially ambiguous WFA iff it terminates on every sequence of the form $(vw^k)_{k\geq 1}$.

References

[1] D. Kirsten. A Burnside approach to the termination of Mohri's algorithm for polynomially ambiguous min-plus-automata. (Manuscript, available under www.math.tu-dresden.de/~kirsten/publications/), 2005.

[2] M. Mohri. Finite-state transducers in language and speech processing. *Computational Linguistics*, 23:269–311, 1997.

Preservation of Recognizability for o-substitution

Andreas Maletti

Recognizable tree series (see [1] and references provided therein) received a lot of attention recently. In [4, 2, 3] an automaton model, called tree series transducer, which transforms tree series was introduced. With this model we can (finitely) represent tree series transformations. A tree series transducer is called recognizable, if all tree series in its tree representation are recognizable [4]. In [4] it was shown that nondeleting and linear recognizable top-down tree series transducers preserve recognizability of tree series in commutative and continuous semirings; *i.e.*, if the input tree series is recognizable then the transformed tree series is also recognizable. In this contribution we consider the question whether o-substitution [3] preserves recognizability of tree series. In part, this also answers the question of preservation of recognizability for pure substitution [2].

First we show that o-substitution preserves recognizable tree series, provided that the semiring is commutative, additively idempotent, and continuous and the target tree series (into which we substitute) is linear (in the substitution variables). The o-substitution is at the heart of the o-tree-series-to-tree-series semantics of tree series transducers [3], so that we consider preservation of recognizability for o-tree-series-to-tree-series transformations computed by linear recognizable bottom-up tree series transducers over commutative, additively idempotent, and continuous semirings. Together with composition results for linear bottom-up tree series transducers this contributes to the theory of abstract families of tree series [5].

References

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Recognizable skew formal power series over trace monoids

Christian Mathissen
Institut für Informatik, Universität Leipzig
D-04009 Leipzig, Germany

mathissen@informatik.uni-leipzig.de

In 2002 Droste and Kuske proposed new semantics for weighted finite automata over words. The transition costs are calculated applying endomorphisms dependent on the word having been read before [DK03]. The behavior is described by so-called skew formal power series. These series are a generalization of conventional formal power series where now the Cauchy–product takes the above mentioned endomorphisms into account.

We extend these series to trace monoids. Trace monoids are free partially commutative monoids. They are an important tool to model concurrent processes. We adapt the model of weighted finite automata over trace monoids in a natural way and investigate their behavior.

Using ideas from [DG99] we achieve a Kleene–Schützenberger theorem, describing recognizable behavior by means of rational expressions. However, the proof that rational series are recognizable does not carry over smoothly. We need restrictions on the monoid generated by certain endomorphisms used in the new semantics.

Counterexamples show that none of the conditions can be dropped.

References

[DG99] Manfred Droste and Paul Gastin. The Kleene–Schützenberger theorem for formal power series in partially commuting variables. *Information and Computation*, 153:47–80, 1999.

[DK03] Manfred Droste and Dietrich Kuske. Skew and infinitary formal power series. In *Proceedings of ICALP'03*, volume 2719 of *Lecture Notes in Computer Science*, pages 426–438. Springer, 2003.

Characterizations of Recognizable Picture Series*

Ina Mäurer
Institut für Informatik, Universität Leipzig
PF 100920, D-04009 Leipzig, Germany
maeurer@informatik.uni-leipzig.de

In the literature, a variety of formal models to recognize or generate two-dimensional objects, called pictures, have been proposed [GR97, IN77]. This research was motivated by problems arising from the area of image processing and pattern recognition [MP69, Fu74], and also plays a role in frameworks concerning cellular automata and other models of parallel computing [LMN98, Smi71]. Restivo and Giammarresi defined the family REC of *recognizable picture languages* (cf. [GR97]). This family is very robust and has been characterized in terms of types of automata, sets of tiles, rational operations or existential monadic second-order (MSO) logic [BG05, GR97, GRST96].

A notion of weighted recognizability for picture languages defined by weighted picture automata (WPA) was introduced in [BG05]. The weights are taken from some commutative semiring. The behavior of a weighted picture automaton is a picture series mapping pictures over an alphabet to some semiring. We will define rational operations and projections on such picture series and generalize devices of tiling systems and 2-dimensional on-line tessellation automata to a quantitative setting involving weights.

Recently, Büchi's fundamental theorem on the coincidence of the class of regular word languages with the family of languages definable in monadic second order logic was generalized to weighted finite automata by introducing a logic with weights [DG05]. Here, we will establish a weighted MSO logic for pictures. The semantics of a weighted formula will be a picture series over a commutative semiring.

We will prove the following theorem [Mäu05, Mäu06]. The required notions will be defined in the talk.

Theorem. Let Σ be an alphabet, K a commutative semiring and $S : \Sigma^{++} \to K$ a picture series. The following assertions are equivalent.

1. S is the behavior of a weighted picture automaton.
2. S is the behavior of a weighted 2-dimensional on-line tessellation automaton.
3. S is the projection of a rational picture series.
4. S is the projection of a tile-local series.
5. S is the semantics of some weighted restricted monadic second-order sentence.

These equivalent weighted picture devices define *recognizable* picture series and can be used to model several application examples, e.g. the intensity of light of a picture (interpreting

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the alphabet as different levels of gray) or the amplitude of a monochrome subpicture of a colored picture. The presented equivalences generalize the main results of [GR97] to a weighted setting.

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Łukasiewicz Logic and Automata over MV-Algebras

Sibylle Schwarz

Institut für Informatik, Martin-Luther-Universität Halle-Wittenberg
email: schwarzs@informatik.uni-halle.de

We associate weighted logics, recently introduced by Droste and Gastin [2], with traditional many-valued logic (see for example [3]). This connection is used to show that for formal series with coefficients in a semiring derived from an MV-algebra, recognizability and definability in a version of second order Łukasiewicz logic coincide.

Definability of formal series in weighted logics was introduced in [2]. Formulas from weighted logic are interpreted in semirings. The interpretation of the connectives in weighted logics exactly reflects the semiring operations and no natural definition of negation is available. Hence syntactically, weighted logic is a (largely) negation-free fragment of MSO-logic on words augmented with the values of the semiring as truth constants. This can be considered as many-valued logic where all predicates are crisp (take truth values only in $\{0, 1\}$).

Many-valued logics are general-purpose extensions of two-valued logic that keep the meaning of the classical connectives as intuitive as possible. Łukasiewicz logic is a well-investigated many-valued logic. The syntax of Łukasiewicz logic uses the classical connectives $\vee, \wedge, \neg$ and quantifiers $\forall, \exists$ as well as the Ł-connectives $\&, \underline{\vee}$. The semantics of Łukasiewicz logic maps sentences to values from the MV-algebra $([0, 1], \oplus, \otimes, \neg, 0, 1)$ where for all $a, b \in [0, 1] : a \oplus b = \min\{1, a + b\}$, $a \otimes b = \max\{0, a + b - 1\}$, and $\neg a = 1 - a$. The truth function for $\underline{\vee}$ is $\oplus$ and the truth function for $\&$ is $\otimes$. The classical connectives $\vee, \wedge$ are interpreted as max and min on $[0, 1]$. On the set $\{0, 1\}$ the meaning of $\underline{\vee}$ and $\vee$ and the meaning of $\&$ and $\wedge$ coincide respectively.

In [1], automata and recognizability over semirings derived by MV-algebras were defined and studied. The semiring $([0, 1], \max, \otimes, 0, 1)$, derived from the MV-algebra $([0, 1], \oplus, \otimes, \neg, 0, 1)$, is commutative, idempotent, ordered, and locally finite. Ł-automata are weighted automata over this semiring. A result from [2] states that for every locally finite commutative semiring K, the set of recognizable series and the set of series definable in weighted logics over K coincide. We apply this result to series recognized by Ł-automata and a version of second order Łukasiewicz logic. In the general proof in [2], for every weighted automaton $\mathcal{A}$ a sentence $\varphi_{\mathcal{A}}$ is constructed such that the semantics of $\varphi_{\mathcal{A}}$ reflects the behavior of $\mathcal{A}$. Since on two-valued formulas (with truth values in $\{0, 1\}$) the truth functions in Łukasiewicz logic coincide with their boolean counterparts, the formula defined by an Ł-automaton is a straightforward extension of the formula in the classical proof [4] by truth constants and simpler than the formula for general semirings presented in [2].

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Observations on the smoothness properties of real functions computed by weighted finite automata

Manfred Droste
Institut für Informatik, Universität Leipzig
PF 100920 D-04009 Leipzig, Germany
droste@informatik.uni-leipzig.de

Jarkko Kari*
Department of Mathematics
FIN-20014 University of Turku, Finland
jkari@utu.fi

Paula Steinby
Turku Center for Computer Science
and Department of Mathematics
FIN-20014 University of Turku, Finland
pauste@utu.fi

We continue investigations of weighted finite automata (WFA) as devices to compute real functions. Based on eigenvalues of the transition matrices of automata we provide a simple necessary condition for continuity and smoothness properties of the functions they compute. Using this condition we show that polynomials are the only smooth functions computed by WFA and that any WFA computing a polynomial of degree k must have at least $k + 1$ states. The results answer some previously posed open problems.

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Parametric Weighted Finite Automata with Unary Alphabet

GERMAN TISCHLER
Lehrstuhl für Informatik II, Universität Würzburg
Am Hubland, 97074 Würzburg, Germany
e-mail: tischler@informatik.uni-wuerzburg.de

ABSTRACT

Weighted finite automata [1] (WFA) are abstract devices that can be used to compute real functions. Parametric weighted finite automata [2],[4] (PWFA) are a variant of WFA with a multidimensional codomain. The set computed by a PWFA is a subset of this codomain. It has been shown in [3] that each set computable by a PWFA is computable by a PWFA with an alphabet cardinality of 2. In this paper we investigate the computational power of PWFA with a unary alphabet. It is known from [2] that unary alphabet PWFA can produce circles, thus in contrast to unary alphabet WFA unary alphabet PWFA can produce non-trivial results. We show that the set of sets computable by automata in this class is just like that for general PWFA closed under the operations of set union, regular restriction of the domain language and invertible affine transformation. We can produce a set of k concentric circles in $\mathbb{R}^2$ with as little as $2k$ states and a set of k circles that have each been subject to different invertible affine transformations with $10k + 1$ states. We discuss which properties the transition matrix has to fulfill to produce non-trivial, that means non-empty and not single point, sets. We also investigate the sets computable by one label PWFA for the case where each produced vector is wrapped component-wise at a certain number.

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Hybridization of near random to random access video compression with weighted finite automata

GERMAN TISCHLER
Lehrstuhl für Informatik II, Universität Würzburg
Am Hubland, 97074 Würzburg, Germany
e-mail: tischler@informatik.uni-wuerzburg.de

ABSTRACT

Video compression in its current form is heavily based on prediction and context dependencies. If we define the term random access by being able to decode one frame by itself without having to completely decode other frames, current standards like those released by the MPEG for the encoding of movies only allow random access to a small fraction of the encoded frames. For MPEG1 this fraction is usually in the order of 1/12, for MPEG4 it is even smaller. This allows random access to the movie at intervals that range from about half a second to a few seconds, which is sufficient for the main application areas of MPEG. However for situations where random access is required, because the delay resulting from having to decode up to 10 or 100 frames until a certain frame is accessible can not be tolerated, another approach has to be used.

We present a near random to random access compression approach for low motion video that is based on hybridization of discrete cosine transform coding, vector quantization and the usual matching pursuit algorithm for weighted finite automata [1],[2],[3]. By near random access we mean that the decoding process per video has an initizialisation time that is in the order of decoding a few frames and after that the decoding is random access.

For the application case where the considered video was generated by artificial 3d-models of spline defined surfaces with textures where the spline control points change during the video ([4], see figure 1) we also discuss the description of these movies by parametric weighted finite automata.

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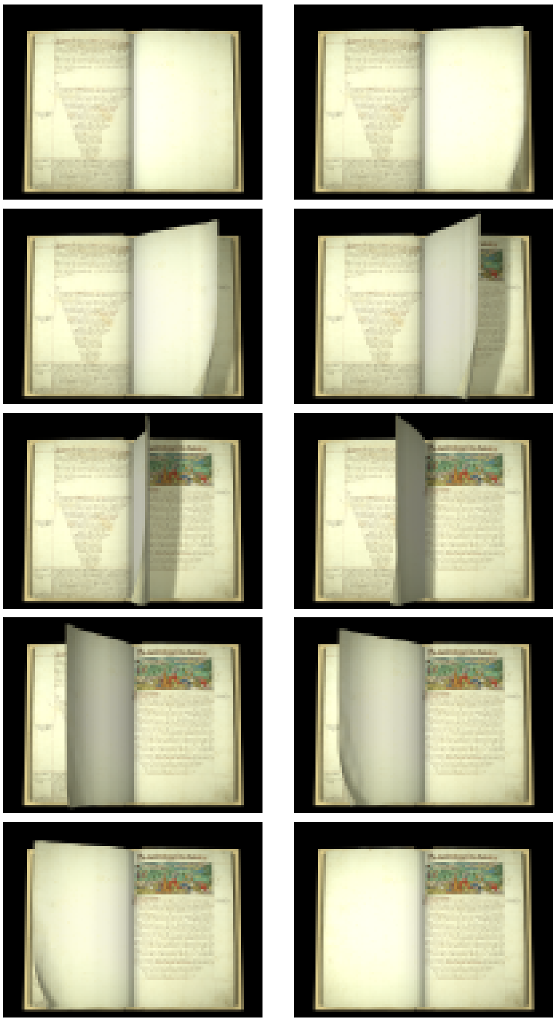


Figure 1: Frames of page-flip video taken from [4].

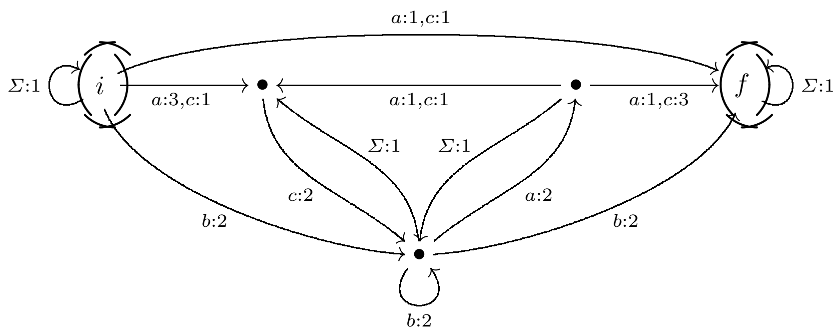
Weighted Automata for Proving Termination of String Rewriting (Extended Abstract)

Johannes Waldmann
waldmann@imn.htwk-leipzig.de

Hochschule für Technik, Wirtschaft und Kultur (FH) Leipzig
Fachbereich IMN, Postfach 30 11 66, D-04251 Leipzig, Germany.

Introduction. Rewriting is a model of computation, and termination is an interesting property of computations. Therefore, proving termination of rewriting is an active area of research [6]. One is particularily interested in termination proofs that can be obtained automatically. We present one new such method, using weighted automata. The idea is that weights should be decreasing when rewriting takes place.

For example, the following $\mathbb{N}$-weighted automaton (where i is the initial state and f is the final state) over $\Sigma = \{a, b, c\}$, that has been found by a computerized search, solves an open problem in string rewriting [7]: The rewriting system $\{aa \to bc, bb \to ac, cc \to ab\}$ is terminating.



Main idea. A weighted automaton A assigns to a word w over an alphabet Σ a value $A(w)$ from some domain (semi-ring) V. Assume V is (strict) partially ordered by $>$. The automaton A is called *compatible* with a relation $\to$ on Σ^* if $\forall x, y \in \Sigma^* : (x \to y) \Rightarrow A(x) > A(y)$. Now if $(V, >)$ is well-founded, and A is compatible with $\to$, then $(\Sigma^*, \to)$ is well-founded, too. We take $\to$ as the rewrite relation $\to_R$ generated by some string rewriting system R over Σ. Then a compatible automaton A for $\to_R$ proves (certifies) termination of R.

We use automata A of the following shape:

- There is exactly one initial state i and exactly one final state f, and for each letter $x \in \Sigma$ we have $A(i, x, i) > 0$ and $A(f, x, f) > 0$ where $A(p, w, q)$ denotes the weight of the word w when A reads it from state p to q.
- For each pair (p, q) of states and each rule $(l \to r)$ of the rewriting system R, we have $A(p, l, q) \geq A(p, r, q)$.

– For each rule $(l \to r) \in R$, we have $A(i, l, f) > A(i, r, f)$.

If the semiring V of the automaton is such that the strict part of the ordering is preserved by addition and by multiplication with a non-zero element, then such an automaton is compatible with R.

Implementation. The obvious choice for the evaluation domain V is the well-founded semi-ring of natural numbers with the standard addition, multiplication and ordering. Using that semi-ring, we obtain an automated method for proving termination of string rewriting. Using a different semi-ring, we draw a connection to the concept of *match bounds* [5], also used in automated proofs of termination.

In our implementation, a compatible $\mathbb{N}$-weighted automaton is found by a finite domain constraint solver (hand-written for this purpose) that translates the problem into a propositional logic formula for which a satisfying assignment is found by the SAT solver SateliteGTI [4].

This method, in concert with match bounds, turns out to be surprisingly successfull when compared to other methods for automated proofs of termination, as shown by the performance of our prover on the Termination Problem Data Base [1].

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