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Programming with Gaigen 2

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Overview Part I

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(and want my program to be reasonably efficient?)

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- A lot of example code.

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- A lot of example code.
- Some exercises with solutions.

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- C++ GA implementations generated by `Gaigen 2`.
- Extra utility code.
- A lot of example code.
- Some exercises with solutions.
- (comes with a book, too).

Overview Part II

Part II: `Gaigen 2`: a Geometric Algebra Implementation Generator.

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- Ideas behind `Gaigen 2`.

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- Generating the code.
- Profiling.

Part I: GA Sandbox

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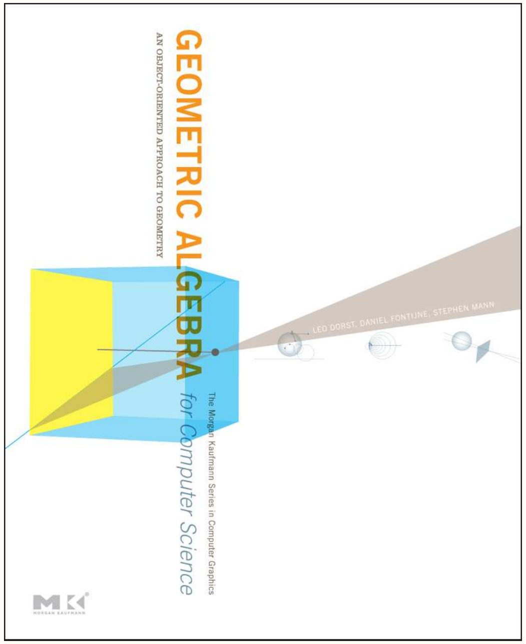
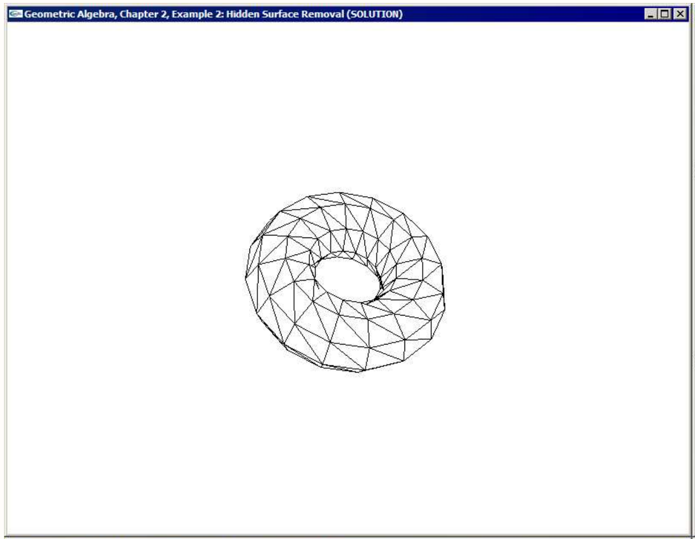
GA Sandbox Overview

`GA Sandbox` was created as a software companion to *Geometric Algebra for Computer Science*.

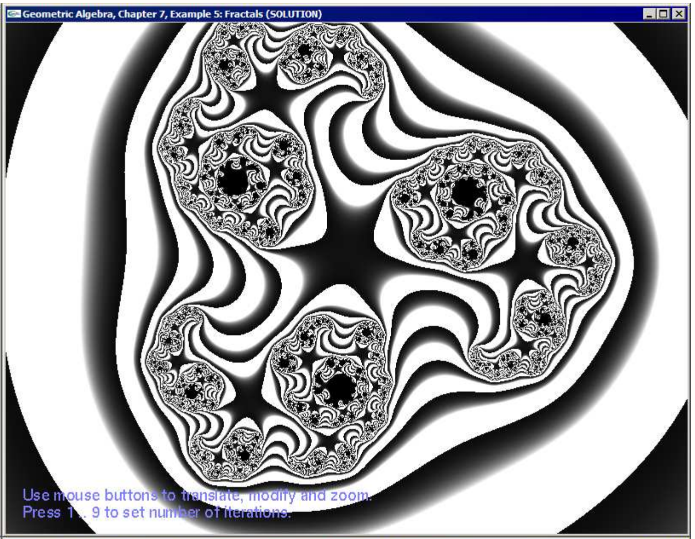

GEOMETRIC ALGEBRA for Computer Science
AN OBJECT-ORIENTED APPROACH TO GEOMETRY
LEO DORST, DANIEL FONTIJNE, STEPHEN MANN
The Morgan Kaufmann Series in Computer Graphics
MK
MORGAN KAUFMANN

You may download `GA Sandbox` at:
http://www.geometricalgebra.net/sandbox.html
or on the handout CD in directory `/gasandbox`

GA Sandbox Examples

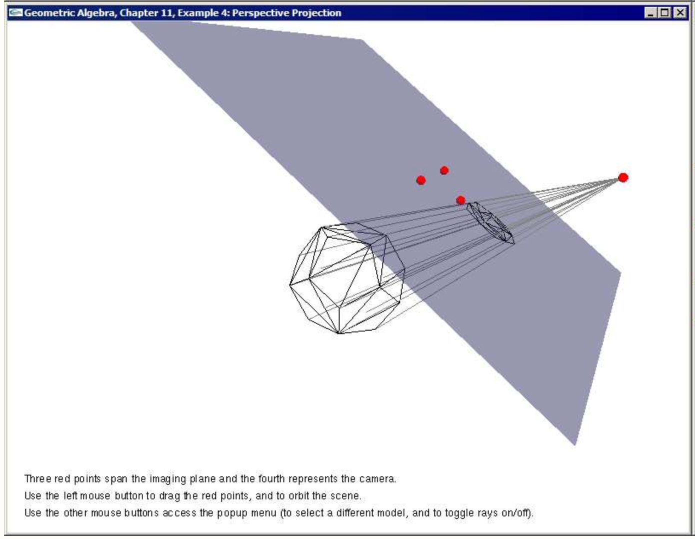


Geometric Algebra, Chapter 2, Example 2: Hidden Surface Removal (SOLUTION)



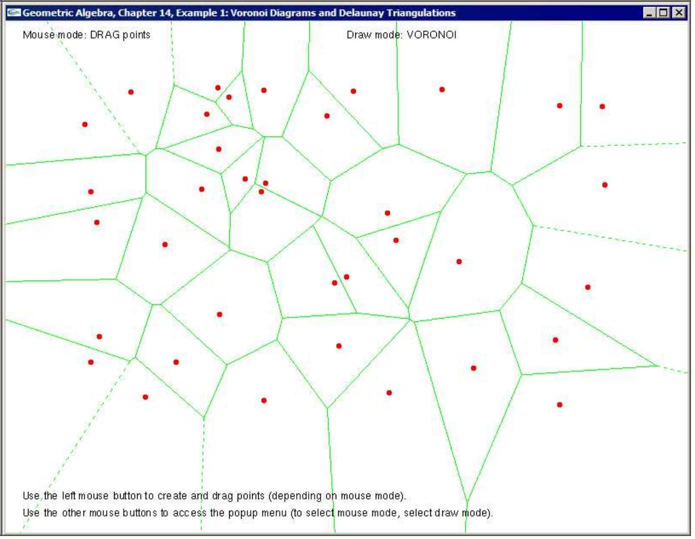
Geometric Algebra, Chapter 7, Example 5: Fractals (SOLUTION)

Use mouse buttons to translate, modify and zoom.
Press 1 .. 9 to set number of iterations.



Geometric Algebra, Chapter 11, Example 4: Perspective Projection

Three red points span the imaging plane and the fourth represents the camera.
Use the left mouse button to drag the red points, and to orbit the scene.
Use the other mouse buttons access the popup menu (to select a different model, and to toggle rays on/off).



Geometric Algebra, Chapter 14, Example 1: Voronoi Diagrams and Delaunay Triangulations

Mouse mode: DRAG points

Draw mode: VORONOI

Use the left mouse button to create and drag points (depending on mouse mode).
Use the other mouse buttons to access the popup menu (to select mouse mode, select draw mode).

Building the Source Code

Linux:

- Install libraries (`GLUT`, `ANTLR`, `OpenCV`, `FLTK`).
- Extract `GA Sandbox` package.
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Windows / Visual Studio:

- Install libraries (or extract precompiled libraries from zip).
- Extract `GA Sandbox` package.
- Open project and build it.

GA Sandbox Contents

Contents of `GA Sandbox`:

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- (library `QHull` for computing convex hulls).

libgasandbox 1/2

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- Basis vectors, like e1, e2, e3.
- Other constants, like I.
- Basic functions: products, inversion, dualization, . . .
- Operators:

  outer product: $\wedge$

  geometric product: *

  left contraction (an inner product): $<<$

libgasandbox 2/2

Other files in `libgasandbox`:

`xxga_util.cpp`: utility functions.
`xxga_draw.cpp`: `OpenGL` drawing functions.
`mv_analyze_xxga.cpp`: multivector ‘analysis’ functions.

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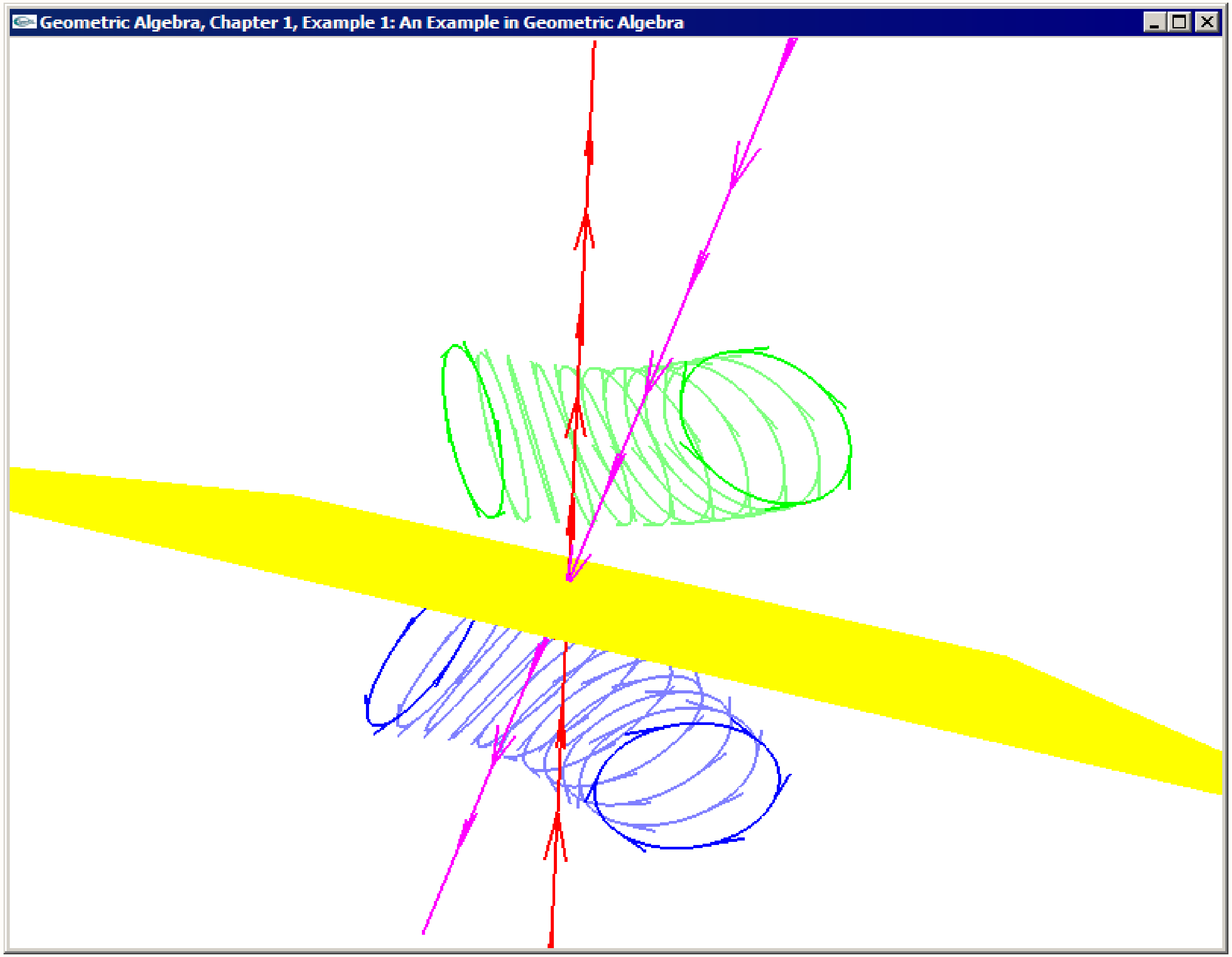
Chapter 1 Example Code



Geometric Algebra, Chapter 1, Example 1: An Example in Geometric Algebra

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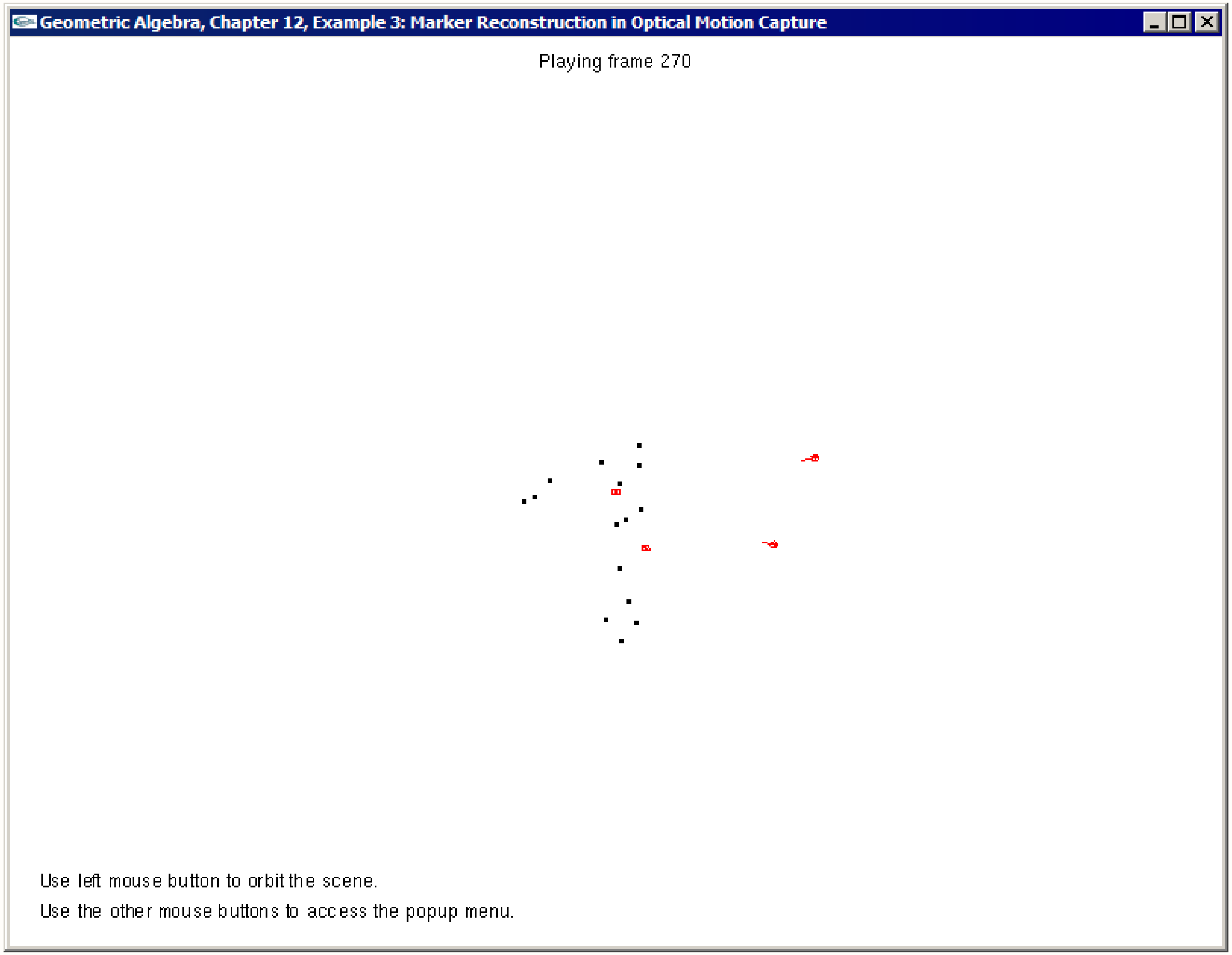
Optical Motion Capture Example Code



Geometric Algebra, Chapter 12, Example 3: Marker Reconstruction in Optical Motion Capture

Playing frame 270

Use left mouse button to orbit the scene.

Use the other mouse buttons to access the popup menu.

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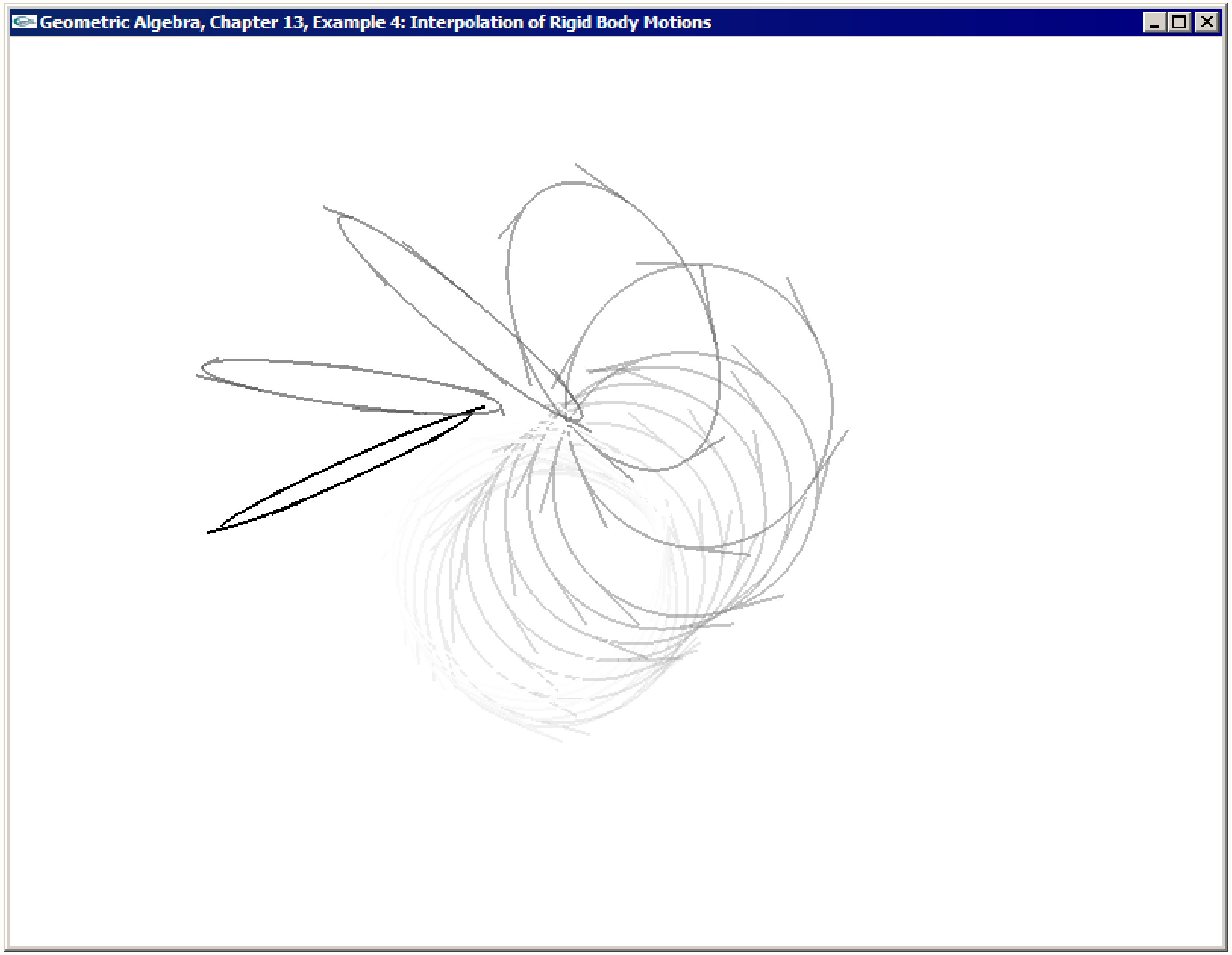
Translation / Rotation Interpolation Code



Geometric Algebra, Chapter 13, Example 4: Interpolation of Rigid Body Motions

Part II: Gaigen 2

Part II: `Gaigen 2`

Gaigen 2 Overview 1/2

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Input: specification of a geometric algebra + optional profile.

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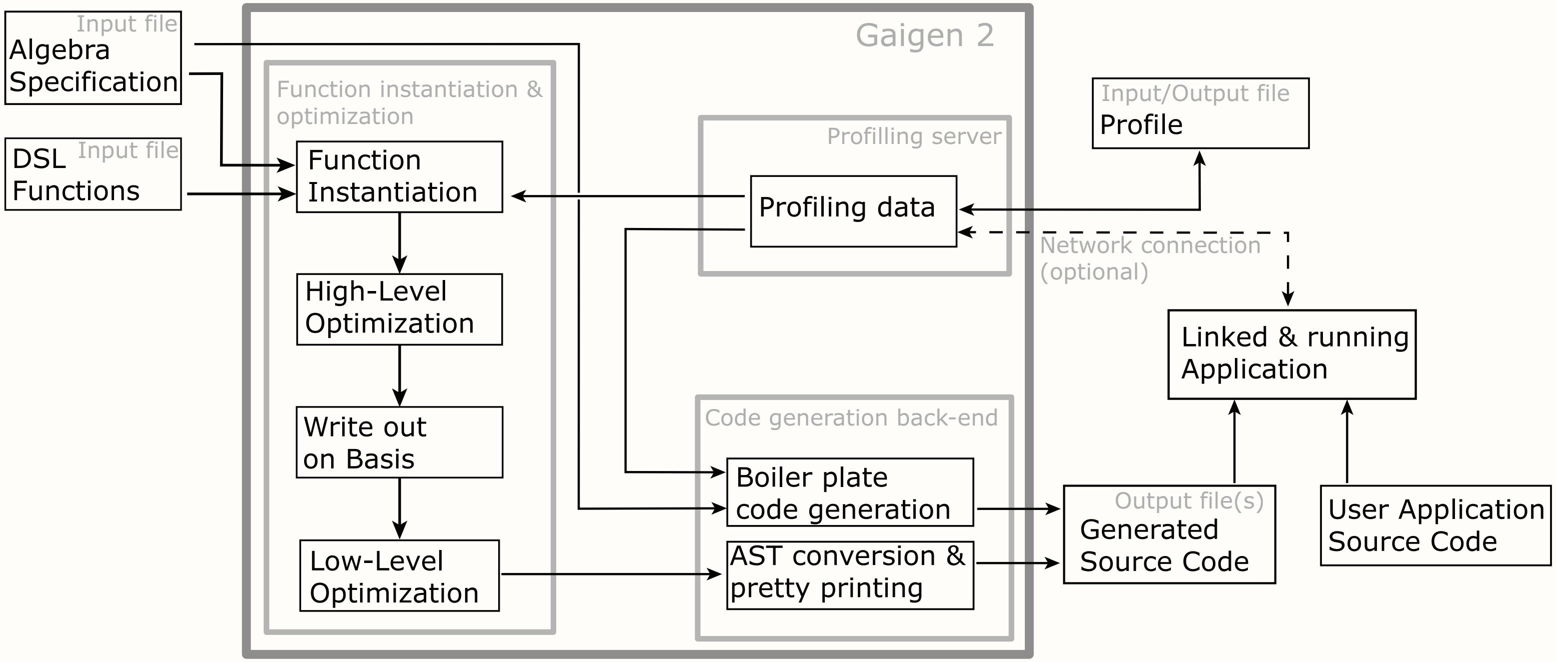

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But: personally I used it successfully for several larger projects:

- Set of simple ray tracers.
- `GA Sandbox`.
- Optical motion capture system.

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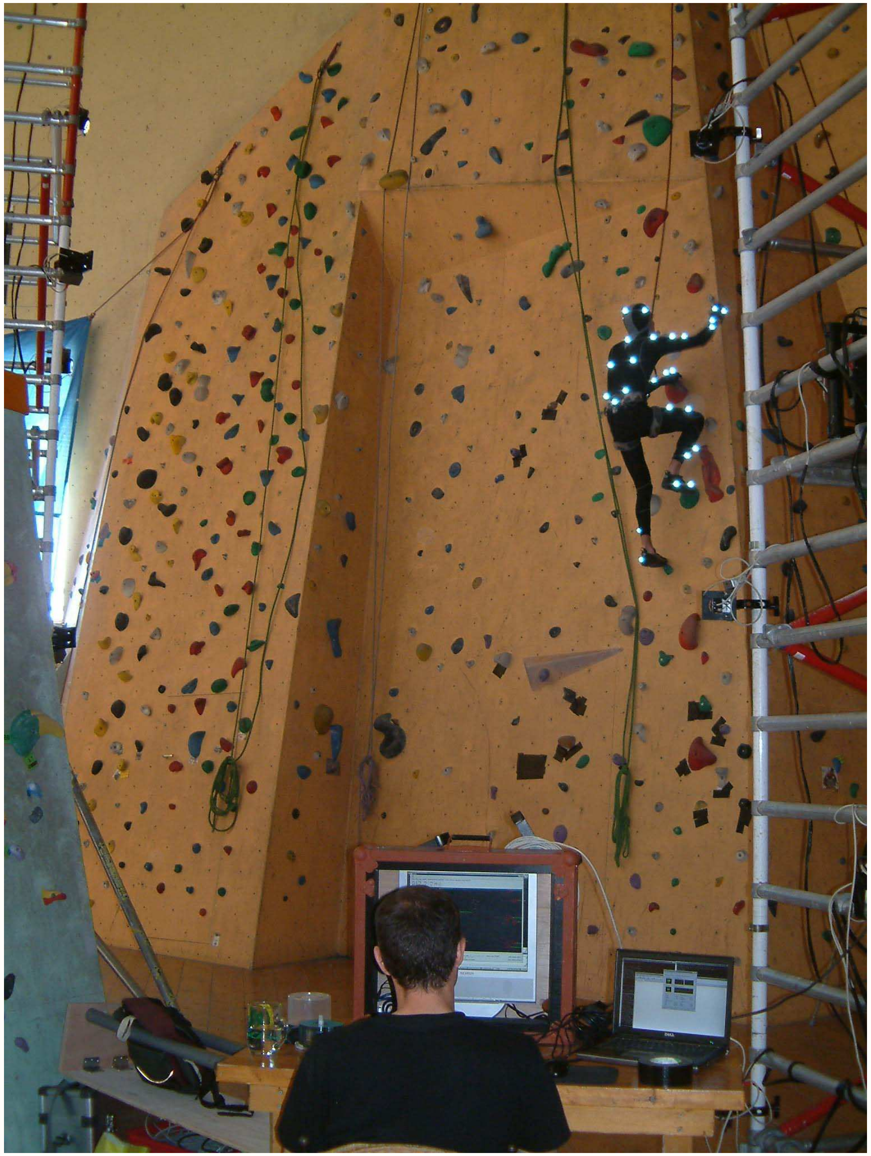
Gaigen 2 used for Motion Capture



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Ray tracer benchmarks

model	implementation	rendering time
3D LA	standard	1.00×
4D LA	standard	1.22×
3D GA	Gaigen 2	0.98×
4D GA	Gaigen 2	1.2×
5D GA	Gaigen 2	1.26×
3D GA	Gaigen 1	2.56×
4D GA	Gaigen 1	2.97×
5D GA	Gaigen 1	5.71×
3D GA	CLU	78×
5D GA	CLU	178×

Multivectors Representation in Gaigen 2

Like many other GA implementations, `Gaigen 2` represents multivectors as a sum of basis blades.

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Example of basis for 3-D space:

$$\{\ \underbrace{1}_{\textit{grade } 0}\ ,\ \underbrace{\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3}_{\textit{grade } 1}\ ,\ \underbrace{\mathbf{e}_1 \wedge \mathbf{e}_2,\ \mathbf{e}_2 \wedge \mathbf{e}_3,\ \mathbf{e}_1 \wedge \mathbf{e}_3}_{\textit{grade } 2}\ ,\ \underbrace{\mathbf{e}_1 \wedge \mathbf{e}_2 \wedge \mathbf{e}_3}_{\textit{grade } 3}\}.$$

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For example, a 3D-rotor:

$$\mathbf{R} = -0.30 - 0.04\,\mathbf{e}_2 \wedge \mathbf{e}_3 + 0.86\,\mathbf{e}_3 \wedge \mathbf{e}_1 - 0.68\,\mathbf{e}_1 \wedge \mathbf{e}_2$$

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In general:
n-dimensional geometric algebra $\longrightarrow$ 2^n multivector coordinates.

But in real-world usage, many of those coordinates are zero!

Ideas behind Gaigen 2

Observation in 2001: GA implementations are slow.

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Reasons:

- The multivector is too general.
    - Wastes memory / processing time.
    - Solution: specialized multivector types.
    - A compromise between mathematical elegance and computational efficiency.

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- The multivector is too general.
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  - Solution: specialized multivector types.
  - A compromise between mathematical elegance and computational efficiency.
- Functions over multivectors are slow.
  - Custom functions for each type of argument.

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Specialized multivectors

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specialization: vectorE3GA(e1, e2, e3);
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specialization: normalizedPoint(e0 = 1, e1, e2, e3, einf);
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specialization: normalizedPoint(e0 = 1, e1, e2, e3, einf);
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All basis blades which are *not* listed in the specification are assumed to be constant 0. Memory is only allocated for non-constant coordinates.

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Specializing Functions

Example of generated code: equation $\mathbf{P} = \mathbf{C} \cdot \mathbf{S}$.

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```
inline pointPair innerProduct(const dualCircle& C, const sphere& S) {
    return pointPair(
        -C.c[2] * S.c[2] + C.c[4] * S.c[4] - C.c[1] * S.c[1],
        C.c[0] * S.c[1] - C.c[2] * S.c[3] + C.c[5] * S.c[4],
        C.c[1] * S.c[3] + C.c[0] * S.c[2] + C.c[3] * S.c[4],
        C.c[9] * S.c[1] + C.c[8] * S.c[4] - C.c[2] * S.c[0],
        C.c[6] * S.c[4] + C.c[9] * S.c[3] - C.c[0] * S.c[0],
        -C.c[1] * S.c[0] - C.c[9] * S.c[2] + C.c[7] * S.c[4],
        -C.c[4] * S.c[0] + C.c[8] * S.c[2] + C.c[7] * S.c[1],
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        C.c[5] * S.c[2] - C.c[4] * S.c[3] - C.c[3] * S.c[1]);
}
```

Gaigen 2 Demo

- Installing.
- Writing a specification for an algebra.
- Generating the code.
- Walkthrough of default generated code.
- Profiling.
- The profile.
- Walkthrough of optimized code.

Conclusion / Discussion

Gaigen 2: efficient, usable for big projects, but a bit rough on the edges.

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Possibly will develop Gaigen 2.5: simpler, cleaner, more pragmatic.

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Is code generation always required for maximum efficiency? (Dietmar Hildenbrand is also moving in that direction)