Hyper-minimization for deterministic weighted tree automata

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May 29, 2014

Overview

Weighted Tree Language

- Assigns weight (e.g. a probability) to each tree
- Weight drawn from commutative semiring; e.g. $(\mathbb{Q}, +, \cdot, 0, 1)$

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- Defines the recognizable weighted tree languages

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Application

- Re-ranker for parse trees
- Representation of parses

large models

Basics

Semiring

Definition

A commutative semiring is an algebraic structure $\mathcal{A} = (A, +, \cdot, 0, 1)$

- $(A, +, 0)$ commutative monoid
- $(A, \cdot, 1)$ commutative monoid
- $\cdot$ distributes over $+$ $\quad a \cdot (a_1 + a_2) = (a \cdot a_1) + (a \cdot a_2)$
- $0 \cdot a = 0$ for all $a \in A$

Examples: $(\mathbb{N}, +, \cdot, 0, 1)$ and $(\mathbb{Q}, +, \cdot, 0, 1)$

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Definition

A commutative semifield is a commutative semiring $\mathcal{A} = (A, +, \cdot, 0, 1)$

- for all $a \in A \setminus \{0\}$ there exists $a^{-1} \in A$ with $a \cdot a^{-1} = 1$

Example: $(\mathbb{Q}, +, \cdot, 0, 1)$

Syntax

Definition

Weighted tree automaton (WTA) is tuple $(Q, \Sigma, \mathcal{A}, F, \mu)$ where

- finite set Q *states*
- ranked alphabet Σ *input symbols*
- commutative semiring $\mathcal{A} = (A, +, \cdot, 0, 1)$ *weight structure*
- $F \subseteq Q$ *final states*

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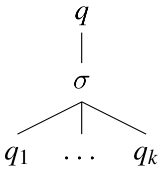
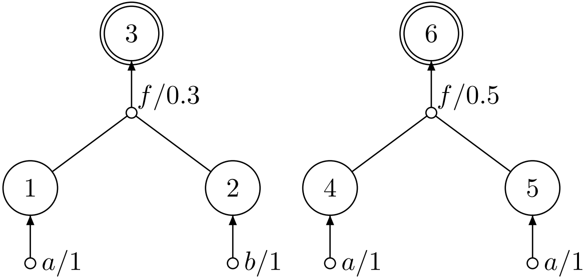
Sample Transition



with weight $\mu_k(\sigma)_{q, q_1 \cdots q_k}$

Syntax — Illustration

Sample Automaton



Semantics

Definition

Let $t \in T_\Sigma(Q)$ and $W = \mathrm{pos}(t)$.

- Run on t: map $r\colon W \to Q$ with $r(w) = t(w)$ if $t(w) \in Q$

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- **Weight** of r

$$\mathrm{wt}(r) = \prod_{\substack{w \in W \\ t(w) \in \Sigma}} \mu_k(t(w))_{r(w), r(w1) \cdots r(wk)}$$

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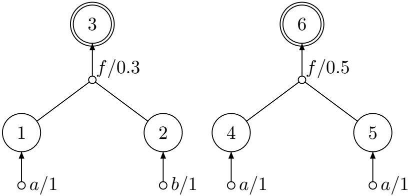
- Recognized weighted tree language

$$\|M\|(t) = \sum_{\substack{r \text{ run on } t \\ r(\varepsilon) \in F}} \mathrm{wt}(r)$$

Semantics — Illustration

Sample Automaton

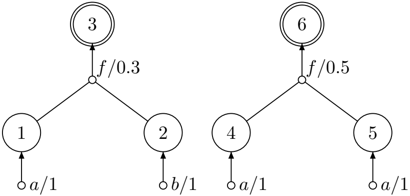


Sample Runs

Input tree: $f(a, b)$ Runs: $6(1, 2)$ with weight 0

Semantics — Illustration

Sample Automaton



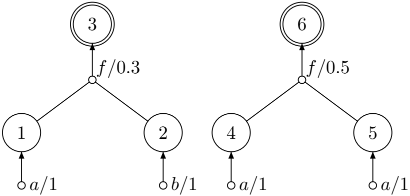
Sample Runs

Input tree: f Runs: f with weight

a b a b

Semantics — Illustration

Sample Automaton

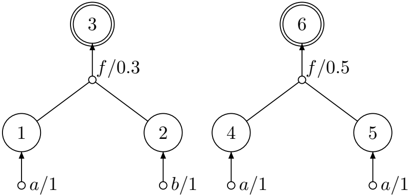


Sample Runs

Input tree: $f(a, b)$ Runs: $f(1, b)$ with weight 1

Semantics — Illustration

Sample Automaton



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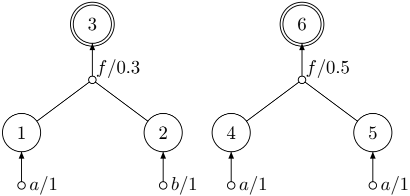

Sample Runs

Input tree: $f(a, b)$ Runs: $3(1, 2)$ with weight 0.3

Determinism

Definition

Deterministic WTA: for every $\sigma \in \Sigma_k$ and $w \in Q^k$ there exists exactly one $q \in Q$ such that $\mu_k(\sigma)_{q,w} \neq 0$

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Notes

- Deterministic WTA does not use addition

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- Determinization possible in locally-finite semirings

[Borchardt, Vogler 2003]

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- Recognizable $\neq$ deterministically recognizable
- Determinization possible in locally-finite semirings [Borchardt, Vogler 2003]
- Partial determinization for probabilities [May, Knight 2006]
- Systematic presentation [Büchse, Vogler 2009]

Hyper-minimization

Assumption

We assume a commutative semifield $\mathcal{A} = (A, +, \cdot, 0, 1)$

Minimization

equivalent = same recognized weighted tree language

Problem

Given deterministic WTA, return

- equivalent deterministic WTA such that
- no equivalent deterministic WTA is smaller

Minimization

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Given deterministic WTA, return

- equivalent deterministic WTA
- minimal

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Problem

Given deterministic WTA, return

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- minimal

Theorem (M., Q. 2011)

Minimization of deterministic WTA can be done in time $\mathcal{O}(m \log n)$

- m = *size of automaton*
- n = *number of states*

Minimization

context = tree with exactly one occurrence of special symbol $\Box$
$c[t]$ = tree obtained from context c by replacing $\Box$ by t

Definition

States p and q are equivalent if there exists $a \in A \setminus \{0\}$ such that

$$\|M\|(c[p]) = a \cdot \|M\|(c[q])$$

for all contexts $c \in C_\Sigma$

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Theorem (Borchardt 2003)

A trim deterministic WTA is minimal
$\iff$ *no pair of different, but equivalent states*

Hyper-minimization

Definition

Languages L and L' almost equal if L and L' have finite difference

$$(L \setminus L') \cup (L' \setminus L)$$

Problem [Badr et al. 2009]

Given DFA, return

- DFA recognizing almost equal language such that
- no smaller DFA recogizes an almost equal language

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Theorem (Holzer, M. 2009, Gawrychowsky, Jeż 2009)

DFA hyper-minimization can be done in time $\mathcal{O}(n \log n)$

Weighted hyper-minimization

$\mathrm{supp}(\tau) = \{t \in T_\Sigma \mid \tau(t) \neq 0\}$ for $\tau \colon T_\Sigma \to A$

Three variants

Two weighted tree languages $\tau_1, \tau_2 \colon T_\Sigma \to A$ are almost equal if

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reduces to the unweighted case

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- q-run = non-zero weighted run with root label q
- preamble state q = finitely many q-runs

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Theorem

A minimal deterministic WTA is hyper-minimal
$\iff$ *no pair of different, but almost equivalent states*
of which one is a preamble state

Algorithm

Overview

Hyper-minimization algorithm

1. Minimize $\mathcal{O}(m \log n)$
2. Compute preamble states $\mathcal{O}(m)$
3. Compute co-preamble states $\mathcal{O}(m)$
4. Identify almost equivalent states $\mathcal{O}(m \log n)$
5. Merge preamble states that are almost equivalent to another state $\mathcal{O}(m)$

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Co-preamble state q = finitely many $c \in C_\Sigma$ such that $\|M\|(c[q]) \neq 0$

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Transition context c is of the shape $\sigma(t_1, \ldots, t_k)$ with

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- exactly one $\Box$ occurs

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Assumptions

- total order on transition contexts

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Assumptions

- total order on transition contexts
- c_q smallest transition context such that $c_q[q]$ evaluates to a co-kernel (i.e., not a co-preamble) state
  for each $q \in Q$

Signature

Definition

Signature of q: $\{\langle c, q', a' \rangle \mid \cdots\}$

- c = transition context
- q' = evaluation of $c[q]$
- a' = transition weight of $c[q]$

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Definition

Standardized signature of q: $\{\langle c, q', a' \rangle \mid q' \text{ co-kernel state, } \cdots\}$

- c = transition context
- q' = evaluation of $c[q]$
- a' = transition weight of $c[q]$ “divided by” transition weight of $c_q[q]$

Signature

Lemma

If two states have the same signature, then they are almost equivalent

Signature

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If two states have the same signature, then they are almost equivalent

Lemma

If two different states are almost equivalent,
then there exist two different states that have the same signature

Finding almost equivalent states

Approach

1. Find two different states of equal signature
2. Merge them using a scaling factor
3. Go to 1.

This will merge more states than desired,
but identifies almost equivalent states

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Hyper-minimization algorithm

Theorem

We can hyper-minimize deterministic WTA in time $\mathcal{O}(m \log n)$

Summary

Solved

- hyper-minimization for deterministic WTA over semifields
- almost equality = finitely many trees with different weight

Open

- Error optimization
- Stronger "almost equality"
- Avoiding requirements
  (semifield; commutativity; determinism; etc.)

Thank you!

References

1. Badr, Geffert, Shipman: *Hyper-minimizing minimized deterministic finite state automata*. ITA 43, 2009
2. Borchardt: *The Myhill-Nerode theorem for recognizable tree series*. Proc. DLT 2003
3. Borchardt, Vogler: *Determinization of finite state weighted tree automata*. JALC 8, 2003
4. Büchse, May, Vogler: *Determinization of weighted tree automata using factorizations*. JALC 15, 2010
5. Gawrychowski, Jeż: *Hyper-minimisation made efficient*. Proc. MFCS 2009
6. Holzer, Maletti: *An $n \log n$ algorithm for hyper-minimizing states in a (minimized) deterministic automaton*. Proc. CIAA 2009
7. Maletti, Quernheim: *Pushing for weighted tree automata*. Proc. MFCS 2011
8. May, Knight: *A better n-best list: practical determinization of weighted finite tree automata*. Proc. HLT-NAACL 2006