Pushing for weighted tree automata

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QuantLA

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Outline

Motivation

Minimization

Equivalence testing

Motivation

Toolkit

- for unweighted and weighted tree automata
- support for all standard operations
- here: *minimization* and *equivalence testing*

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Notes

- mainly developed by THOMAS HANNEFORTH (U Potsdam)
- project name: TALib (tree automata library)

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Notes

- mainly developed by THOMAS HANNEFORTH (U Potsdam)
- project name: TALib (tree automata library)
- fitting *talib* (Arabic, “student”), but plural is *taliban*

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Toolkit

- for unweighted and weighted tree automata
- support for all standard operations
- here: *minimization* and *equivalence testing*

Why those?

- difference between unweighted and weighted
- *minimization* essential for large automata
- *equivalence testing* important for sanity checks

Motivation — Minimization

Typical automata

- English BERKELEY parser grammar 153 MB
  (1,133 states and 4,267,277 transitions)
- English EGRET parser grammar 107 MB
- Chinese EGRET parser grammar 98 MB

Semirings

Definition (commutative semiring $(S, +, \cdot, 0, 1)$)

- commutative monoids $(S, +, 0)$ and $(S, \cdot, 1)$
- absorption $s \cdot 0 = 0$ $\qquad s \in S$
- distributivity $s_1 \cdot (s_2 + s_3) = (s_1 \cdot s_2) + (s_1 \cdot s_3)$ $\qquad s_1, s_2, s_3 \in S$

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Definition (commutative field $(S, +, \cdot, 0, 1)$)

- commutative semifield $(S, +, \cdot, 0, 1)$
- for every $s \in S$ there exists $(-s) \in S$ such that $s + (-s) = 0$

Weighted tree automaton

Definition (wta)

Weighted tree automaton (Q, Σ, μ, F)

- Q finite set of *states*
- Σ ranked alphabet of *input symbols*
- $\mu = (\mu_k)_{k \in \mathbb{N}}$ with $\mu_k \colon \Sigma_k \to S^{Q^k \times Q}$ *transition weight assignment*
- $F \subseteq Q$ *final states*

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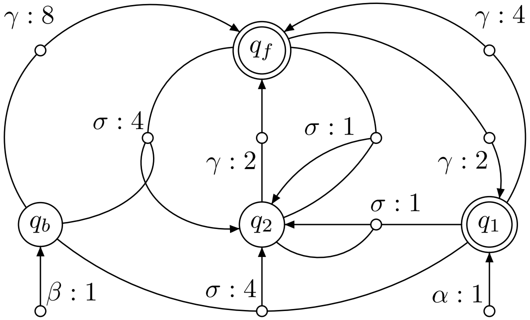
Definition (dwta)

A wta (Q, Σ, μ, F) is deterministic if for all $\sigma \in \Sigma_k$, $q, q', q_1, \dots, q_k \in Q$
$\mu_k(\sigma)_{q_1 \cdots q_k, q} \neq 0 \neq \mu_k(\sigma)_{q_1 \cdots q_k, q'}$ implies $q = q'$

Deterministic weighted tree automaton



Weighted tree automaton

Definition

The semantics of a wta $M = (Q, \Sigma, \mu, F)$ is $M \colon T_\Sigma \to S$

$$M(t) = \sum_{q \in F} h_\mu(t)_q \qquad t \in T_\Sigma$$

with $h_\mu \colon T_\Sigma(Q) \to S^Q$ $\qquad q, q' \in Q$, $\sigma \in \Sigma_k$, $t_1, \dots, t_k \in T_\Sigma(Q)$

$$h_\mu\big(q'\big)_q = \begin{cases} 1 & \text{if } q = q' \\ 0 & \text{otherwise} \end{cases}$$

$$h_\mu\big(\sigma(t_1, \dots, t_k)\big)_q = \sum_{q_1, \dots, q_k \in Q} \mu_k(\sigma)_{q_1 \cdots q_k, q} \cdot \prod_{i=1}^{k} h_\mu(t_i)_{q_i}$$

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State-of-the-art (tree / string)

	unweighted	weighted (field)
dwta	$\mathcal{O}(m \log n)$	$\mathcal{O}(mn)$ / $\mathcal{O}(m \log n)$
wta	PSPACE-complete	P / $\mathcal{O}(mn^2)$

Notes

- unweighted = weighted over $(\{0,1\}, \max, \min, 0, 1)$
- m = size of the transition table
- n = number of states

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Minimization

Let $(S, +, \cdot, 0, 1)$ be a semifield

Definition (Borchardt 2003)

States $q_1, q_2 \in Q$ in dwta (Q, Σ, μ, F) are equivalent ($q_1 \equiv q_2$) if there exists $s \in S \setminus \{0\}$ such that

$$\sum_{q \in F} h_\mu(c[q_1])_q = s \cdot \sum_{q \in F} h_\mu(c[q_2])_q \qquad c \in C_\Sigma(Q)$$

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Notes

- $q_1 \equiv q_2$ if they behave equally in all contexts (up to a constant invertable scaling factor)
- $\equiv$ is a congruence
- finer than the classical (unweighted) state equivalence

Minimization

Theorem (M. 2009)

Dwta over semifields can be minimized in time $\mathcal{O}(mn)$

Approach

1. Compute sign of life for each state
2. Compute equivalence pairwise with scaling factor
3. Merge equivalent states

Minimization

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- $c \in C_\Sigma(Q)$ is sign of life for $q \in Q$ if $\sum_{q' \in F} h_\mu(c[q])_{q'} \neq 0$

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Approach

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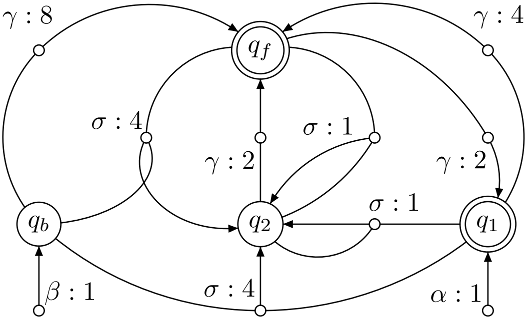
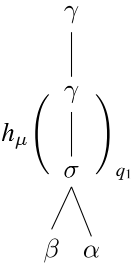
Definition

- $c \in C_\Sigma(Q)$ is sign of life for $q \in Q$ if $\sum_{q' \in F} h_\mu(c[q])_{q'} \neq 0$
- state that has a sign of life is live
- state without a sign of life is dead

Sign of life


$\gamma : 8$
$\gamma : 4$
q_f
$\sigma : 4$
$\sigma : 1$
$\gamma : 2$
$\gamma : 2$
$\sigma : 1$
q_b
q_2
q_1
$\beta : 1$
$\sigma : 4$
$\alpha : 1$


γ
h_μ
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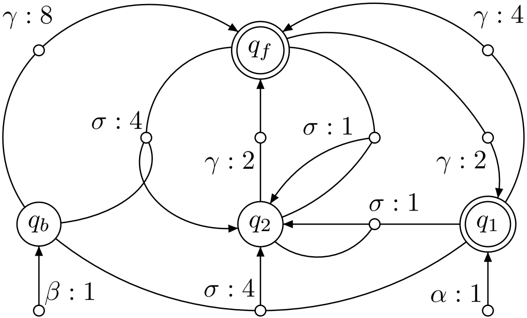
α

Sign of life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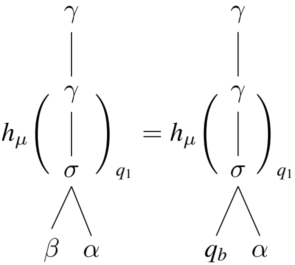
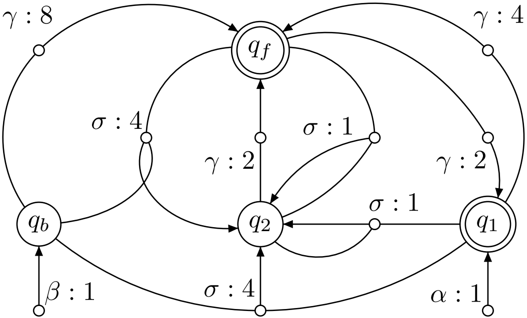

h_μ
γ
γ
σ
β α
q_1
$= h_\mu$
γ
γ
σ
q_b α
q_1

Sign of life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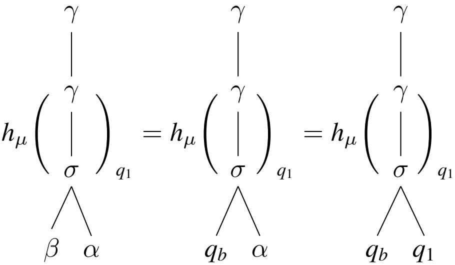
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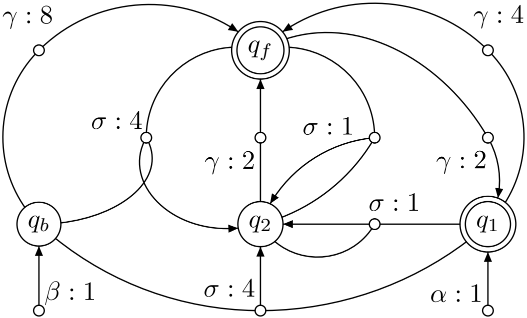
Sign of life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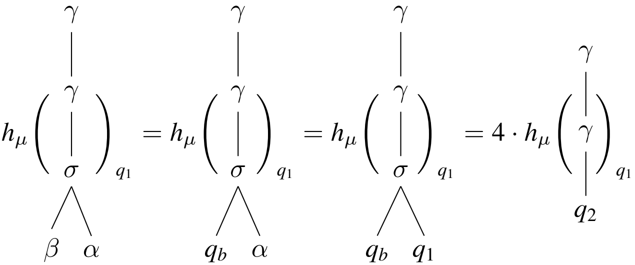

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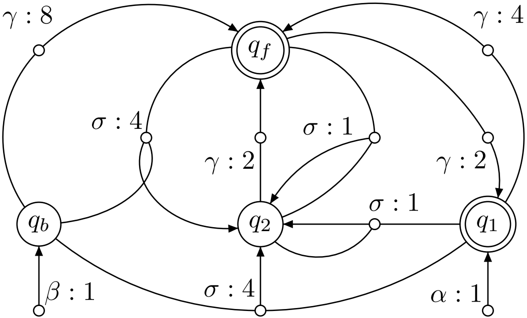
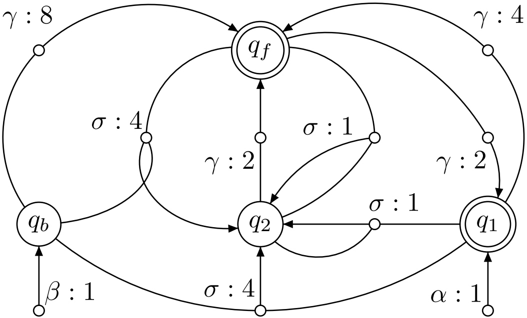


$$h_\mu\big(\gamma(\gamma(\sigma(\beta,\alpha)))\big)_{q_1} = h_\mu\big(\gamma(\gamma(\sigma(q_b,\alpha)))\big)_{q_1} = h_\mu\big(\gamma(\gamma(\sigma(q_b,q_1)))\big)_{q_1} = 4 \cdot h_\mu\big(\gamma(\gamma(q_2))\big)_{q_1} = 8 \cdot h_\mu\big(\gamma(q_f)\big)_{q_1}$$

Sign of life



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Minimization

Our approach

Compute the classical unweighted equivalence

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Choose *one* sign of life for every equivalence class

Minimization

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Choose	*one* sign of life for every equivalence class
Normalize	transition weights according to signs of life (“pushing”)

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Expand	the labels again

Minimization

Our approach

- **Compute** the classical unweighted equivalence
- **Choose** *one* sign of life for every equivalence class
- **Normalize** transition weights according to signs of life (“pushing”)
- **Join** transition labels and weights
- **Minimize** the obtained unweighted dwta
- **Expand** the labels again

$\Rightarrow$ the resulting dwta is minimal

Signs of life

Computation

1. Compute (unweighted) MYHILL-NERODE equivalence $\cong$

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   $\mathrm{sol}([q])$ is a sign of life for q — live $q \in Q$
3. Compute $\lambda\colon Q \to (S \setminus \{0\})$ such that
   - $\lambda(q) = \sum_{q' \in F} h_\mu(c[q])_{q'}$ with $c = \mathrm{sol}([q])$ — live $q \in Q$
   - $\lambda(q) = 1$ — dead $q \in Q$

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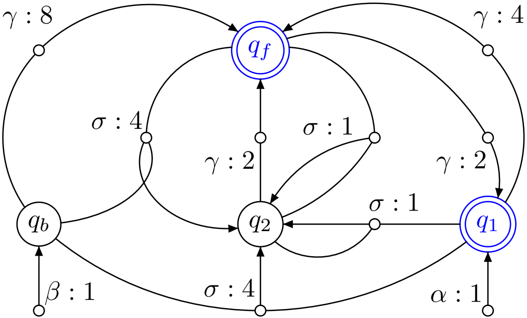
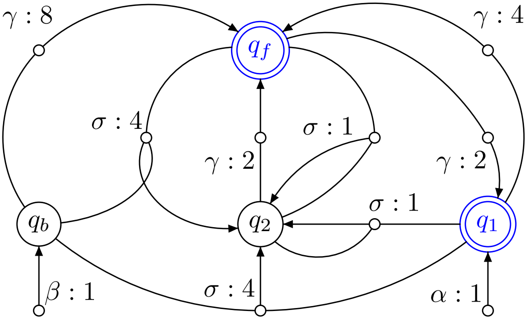
Complexity: $\mathcal{O}(m \log n)$

Signs of life



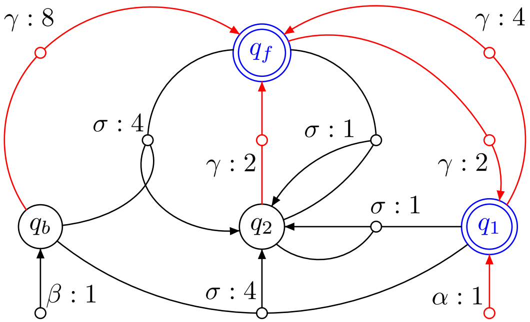
- Start with the equivalence $\cong = \{\{q_1, q_f\}, \{q_2, q_b\}\}$

Signs of life



- Start with the equivalence $\cong = \{\{q_1, q_f\}, \{q_2, q_b\}\}$
- q_1 and q_f are trivially live with $\mathrm{sol}(\{q_1, q_f\}) = \square$
- $\lambda(q_1) = \lambda(q_f) = 1$

Signs of life



- Consider all transitions leading to q_1 or q_f

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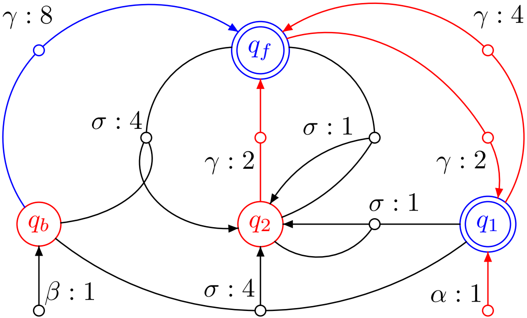

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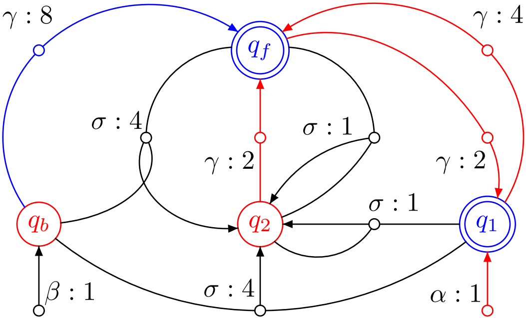

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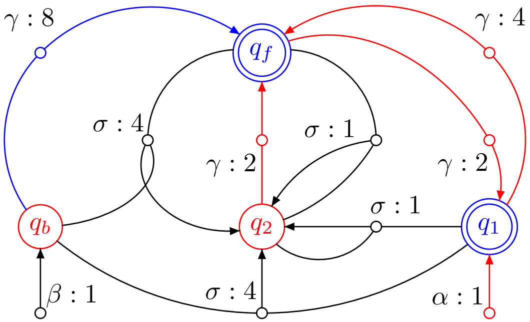

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- Pick $\gamma(q_b)$ because q_b unexplored
- Set $\mathrm{sol}(\{q_b, q_2\}) = \gamma(\square)$
- Set $\lambda(q_b) = \lambda(q_f) \cdot 8 = 8$ and $\lambda(q_2) = \lambda(q_f) \cdot 2 = 2$

Pushing

Definition

Given $\lambda\colon Q \to (S \setminus \{0\})$ such that $\lambda(q) = 1$ for all $q \in F$

$$\mathrm{push}_\lambda(M) = (Q, \Sigma, \mu', F)$$

$$\mu'_k(\sigma)_{q_1 \cdots q_k, q} = \prod_{i=1}^{k} \lambda(q_i)^{-1} \cdot \mu_k(\sigma)_{q_1 \cdots q_k, q} \cdot \lambda(q)$$

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Notes

- Transitions to $q \in Q$ charge additional weight $\lambda(q)$

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Notes

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- Transitions leaving q_i compensate by charging the weight $\lambda(q_i)^{-1}$

Pushing

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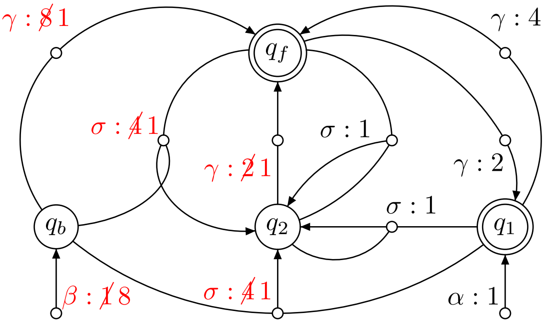
Theorem

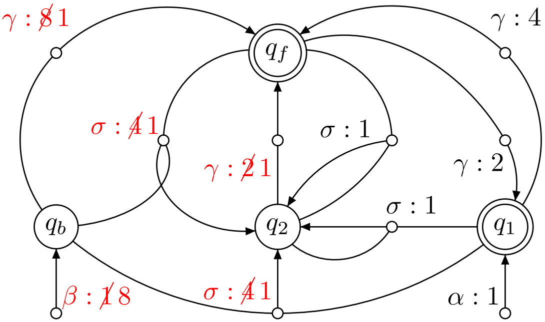
$\mathrm{push}_\lambda(M)$ *and* M *are equivalent*

Pushing



- $\lambda(q_1) = \lambda(q_f) = 1$
- $\lambda(q_2) = 2$
- $\lambda(q_b) = 8$

Pushing

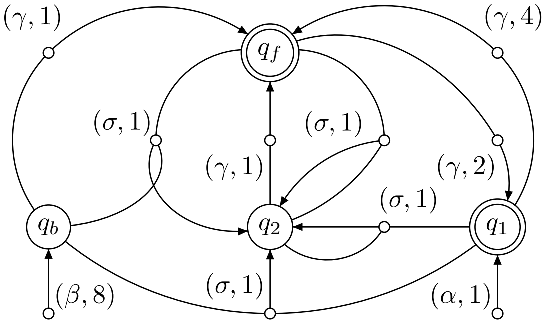


Theorem

Given suitable $\lambda \colon Q \to (S \setminus \{0\})$ *and* $\mathrm{push}_\lambda(M) = (Q, \Sigma, \mu', F)$

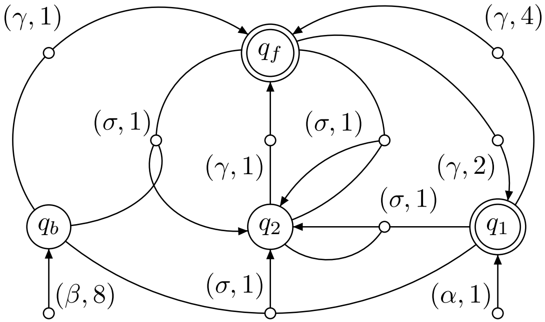
$$\mu'_k(\sigma)_{q_1 \cdots q_k, q} = \mu'_k(\sigma)_{q'_1 \cdots q'_k, q'} \qquad \sigma \in \Sigma_k,\ q_i \equiv q'_i, \text{ and } q \equiv q'$$

Minimization



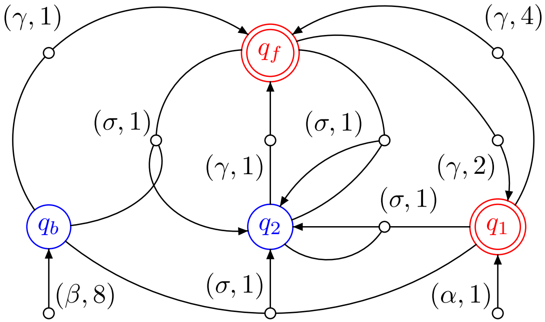
1. Make weight part of the label

Minimization



1. Make weight part of the label
2. minimize as (unweighted) dwta

Minimization



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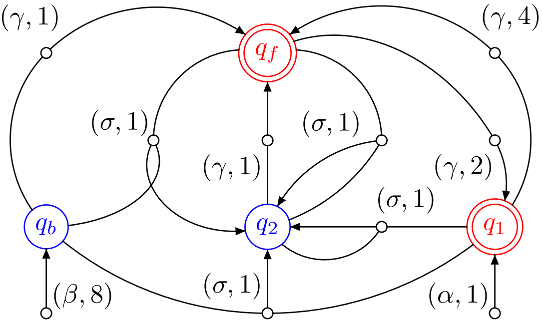
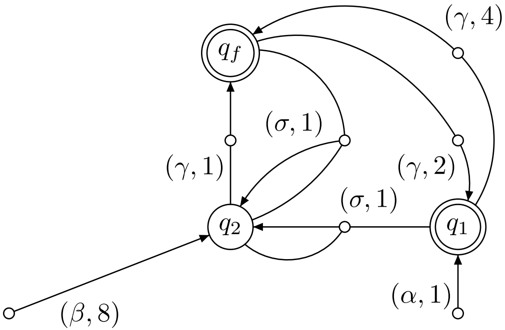
Minimization



1. Make weight part of the label
2. minimize as (unweighted) dwta
   - q_b and q_2 equivalent (will be merged)
   - q_1 and q_f not equivalent

Minimal dwta



Minimization

Theorem

We can minimize dwta in time $\mathcal{O}(m \log n)$

State-of-the-art (tree / string)

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wta	PSPACE-complete	P / $\mathcal{O}(mn^2)$

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Equivalence testing

Equivalence testing — Motivation

Determinization sanity checking

1. Sum (union) construction of dwta M_1 and M_2 yields wta M
2. Determinization yields dwta M'
3. Check equivalence between M' and the result M'' of the union product construction for M_1 and M_2

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Minimization sanity checking

1. Minimize dwta M to obtain M'
2. Check equivalence between M and M'

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~~Minimization sanity checking~~

1. ~~Minimize dwta M to obtain M'~~
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Equivalence testing

State-of-the-art (tree / string)

	unweighted	weighted (field)
dwta	$\mathcal{O}(m \log n)$ / $\mathcal{O}(m \log^* n)$	$\mathcal{O}(m_1 m_2)$ / $\mathcal{O}(m \log n)$
wta	EXPTIME-complete	P / $\mathcal{O}(mn^2)$

Notes

- $m = \max(m_1, m_2)$
- $n = \max(n_1, n_2)$

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Equivalence testing

Definition

Dwta M and M' are push-isomorphic if there exists $\lambda \colon Q \to (S \setminus \{0\})$ with $\lambda(q) = 1$ for all $q \in F$ such that M' is isomorphic to $\mathrm{push}_\lambda(M)$

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All equivalent minimal dwta are push-isomorphic

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Approach

- Minimize both M_1 and M_2

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Theorem

All equivalent minimal dwta are push-isomorphic

Approach

- Minimize both M_1 and M_2
- Check push-isomorphism (isomorphism after special pushing)

Equivalence testing

Theorem

We can test equivalence for dwta in time $\mathcal{O}(m \log n)$

State-of-the-art (tree / string)

	unweighted	weighted (field)
dwta	$\mathcal{O}(m \log n)$ / $\mathcal{O}(m \log^* n)$	$\mathcal{O}(m \log n)$
wta	EXPTIME-complete	P / $\mathcal{O}(mn^2)$

Summary

Minimization (tree / string)

	unweighted	weighted (field)
dwta	$\mathcal{O}(m \log n)$	$\mathcal{O}(m \log n)$
wta	PSPACE-complete	P / $\mathcal{O}(mn^2)$

Equivalence testing (tree / string)

	unweighted	weighted (field)
dwta	$\mathcal{O}(m \log n)$ / $\mathcal{O}(m \log^* n)$	$\mathcal{O}(m \log n)$
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