Qualitative Choice Logic

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Abstract

Qualitative choice logic (QCL) is a propositional logic for representing alternative, ranked options for problem solutions. The logic adds to classical propositional logic a new connective called ordered disjunction: $A \times B$ intuitively means: if possible A, but if A is not possible then at least B. The semantics of qualitative choice logic is based on a preference relation among models. Consequences of QCL theories can be computed through a compilation to stratified knowledge bases which in turn, according to results in [2], can be compiled to classical propositional theories.

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1 Introduction

For many AI applications, e.g. in design or configuration, it is necessary to represent intended properties of a particular problem solution. For instance, if we want to make a hotel booking for a trip to a conference, we intend properties such as being close to the conference site, being close to potential sight-seeing objects, and we want the hotel reasonably priced. Most often not all of the intended properties can be satisfied, that is, we have to make some sort of compromises. To do so it is very convenient to be able to express alternative, second (or third etc.) best options which one would like satisfied if the best option is unavailable. For the hotel booking example, for instance, we may want to express that we prefer to stay within walking distance of the conference site, if that is not possible transportation provided by the hotel should be available, if also this is not possible we want at least public transportation (taxis are not being reimbursed according to our university's travel refund policy).

To represent options of this kind we introduce in this paper a new nonmonotonic propositional logic for representing qualitative choices - hence the name qualitative choice logic (QCL). The logic is different from existing nonmonotonic logics in the way nonmonotonicity is introduced: we do not use non-standard inference rules, as in Reiter's default logic [18], modal operators expressing consistency or belief, as in autoepistemic logic [16], or abnormality predicates whose extensions are minimized, as in circumscription [14, 15]. The non-standard part of our logic is a new logical connective $\times$ which is fully embedded in the logical language. Intuitively, if A and B are formulas then $A \times B$ says: if possible A, but if A is impossible then (at least) B.

The intended use of this logic can be illustrated using the hotel booking example. Assume we want to represent the options concerning the location. Using mnemonic symbols we express the options as follows:

$$walking \times hotel{-}transport \times public{-}transport$$

Assume there are 4 hotels available out of which we have to pick one:

$$hotel_1 \vee hotel_2 \vee hotel_3 \vee hotel_4$$

We have the following information about the hotels

$hotel_1 \rightarrow walking$
$hotel_2 \rightarrow \neg walking \wedge hotel{-}transport$
$hotel_3 \rightarrow \neg walking \wedge \neg hotel{-}transport \wedge public{-}transport$
$hotel_4 \rightarrow \neg walking \wedge \neg hotel{-}transport \wedge \neg public{-}transport$

Given these propositional formulas QCL will give us the conclusion

$$hotel_1$$

since this is the only hotel satisfying our most intended option. Now assume that, after calling the hotel we find out that it is fully booked, that is, we have the additional information

$$\neg hotel_1$$

This means that our most favoured property, being within walking distance of the conference site, cannot be satisfied. We now obtain the conclusion

$$hotel_2$$

which is not exactly what we wanted but better than nothing.

Our new connective $\times$ can be viewed as a kind of disjunction where the order of the disjuncts matters: $A \times B$ is obviously very different form $B \times A$. We therefore call it ordered disjunction. Its semantics will be defined in terms of preferred models.

The rest of the paper is organized as follows: We first introduce syntax and semantics for QCL and give a few motivating examples. We then consider aspects of computation. It turns out that formulas have a special normal form. Theories in this normal form can be translated to stratified knowledge bases. These in turn can, according to the results in [2], be transformed into a classical propositional theory. We thus can compute consequences of QCL through a compilation process. Sect. 4 briefly describes potential applications of QCL. Due to space limitations this section is very short. It will be extended in the final version of the paper. In Sect. 5 we briefly present several alternative definitions of the consequence relation. Sect. 6 concludes the paper.

2 Syntax and semantics of QCL

2.1 Syntax

We start with standard propositional logic and add a new non-standard kind of disjunction: given formulas $A_1, \ldots, A_n$[1] we use

$$A_1 \times \ldots \times A_n$$

[1]Throughout the paper we use capital letters from the beginning of the alphabet to represent formulas.

to express: some A_j must be true, preferably A_1, but if this is not possible then A_2, if this is not possible A_3, etc. The idea is that from $A \times B$ you get A, from $A \times B, \neg A$ you get B ($A \times B, \neg A, \neg B$ is inconsistent). Clearly, the order is what matters. We, therefore, call $\times$ ordered disjunction.

Ordered disjunction is fully embedded in the logical language to make sure that context dependent options can be expressed, that is we may have formulas like $A \rightarrow (B \times C)$ and $\neg A \rightarrow (C \times B)$. The precise definition of the syntax is as follows:

Definition 1 *Let $\mathcal{V}$ be a set of atoms. The set of well-formed formulas of QCL is inductively defined as follows*

1. *every element of $\mathcal{V}$ is a well-formed formula,*
2. *if F_1 and F_2 are well-formed formulas, then $(\neg F_1)$, $(F_1 \vee F_2)$, $(F_1 \wedge F_2)$ and $(F_1 \times F_2)$ are well-formed formulas.*

As usual we use $A \rightarrow B$ as an abbreviation for $\neg A \vee B$. We omit unnecessary brackets. $A_1 \times A_2 \times \ldots \times A_n$ abbreviates $(A_1 \times (A_2 \times (\ldots (A_{n-1} \times A_n) \ldots)))$[2].

2.2 Semantics

The semantics of *QCL* is based on the degree of satisfaction of a formula in a particular (classical) model. Intuitively, the degree can be viewed as a measure of disappointment: the higher the degree the more disappointing the model (or the farther away from complete satisfaction).

As in standard propositional logic, an interpretation I is an assignment of the classical truth values T, F to the atoms. We identify I with the subset of true atoms.

Interpretations can satisfy formulas to a certain degree. For instance, if A and B are atoms and I contains A then I satisfies the formula $A \times B$ to degree 1. If I does not contain A but B, then it satisfies $A \times B$ to degree 2.

To determine the degree of disappointment for a model we need to establish, for each formula F, how many different possible levels of satisfaction the formula provides, in other words, how many potential backups (including the most favourable option) F admits. This is captured in the following notion of optionality.

Definition 2 *The optionality of a formula is defined as follows:*
$opt(A) = 1$ *if A is an atom*
$opt(\neg F) = 1$

[2]We later show that $\times$ is associative, so the brackets do not matter.

$opt(F_1 \vee F_2) = max(opt(F_1), opt(F_2))$
$opt(F_1 \wedge F_2) = max(opt(F_1), opt(F_2))$
$opt(F_1 \times F_2) = opt(F_1) + opt(F_2)$

Intuitively, if the optionality of F is n, then there is a best way, a second best way, etc. and an n-th best way of satisfying F. For classical formulas there is only one way to satisfy them, hence they all have optionality 1. If $F = A \times B$, where A and B are classical, 2 degrees are needed. The best way of satisfying F is making A true, but if this doesn't work there is still the second best option, namely making B true.

The optionality of a negated formula may seem puzzling at first, but there is not more than one way of making $\neg(A \times B)$ true: you must make A and B false, there is no second best solution for this. In a sense negation transforms nested ordered disjunctions into standard disjunctions. Hence $\neg F$ behaves like a classical formula which explains why $opt(\neg F) = 1$.

Note that for formulas of the form $A_1 \times \ldots \times A_n$ where all A_i are classical opt is n.

We next define the satisfaction relation. The relation is indexed according to the degree of satisfaction of a formula in a model.

Definition 3

1. *$I \models_1 A$ iff $A \in I$ (for propositional atoms A)*
2. *$I \models_k P \wedge Q$ iff $I \models_m P$ and $I \models_n Q$ and $k = max(m, n)$*
3. *$I \models_k P \vee Q$ iff*
   $[I \models_k P$ and for no $j < k : I \models_j Q]$ or
   $[I \models_k Q$ and for no $j < k : I \models_j P]$
4. *$I \models_1 \neg P$ iff for no m: $I \models_m P$*
5. *$I \models_k P \times Q$ iff*
   $I \models_k P$ or
   $I \models_1 \neg P$, $I \models_m Q$, and $k = m + opt(P)$.

Let us illustrate the satisfiability relation using a simple example. Let $F = A \vee (B \times C)$ where A, B and C are atoms. Now all interpretations I containing A as well as those containing B satisfy F with degree 1, that is, we have $I \models_1 F$. Now assume I is an interpretation which contains C but makes A and B false. In this case F is satisfied with degree 2, that is $I \models_2 F$. Interpretations satisfying none of the three atoms do not satisfy F to any degree; they satisfy $\neg F$ with degree 1.

Lemma 1 *$I \models_k F$ and $I \models_j F$ implies $k = j$.*

Lemma 2 *$I \models_k F$ implies $k \leq opt(F)$.*

The converse is false. Indeed, let $F = A \vee (B \times C)$ where A, B, C are atoms. Let $I = \{A, B, C\}$. We have $I \models_1 F$, but $opt(F) = max(opt(A), opt(B \times C)) = 2$.

It will be shown later that all formulas containing $\times$ can be transformed equivalently into so-called basic choice formulas of the form $(A_1 \times \ldots \times A_n)$ where the A_i do not contain $\times$. For formulas of this kind clause 5. in the definition of the satisfaction relation can be simplified to

5'. $I \models_k P \times Q$ iff $I \models_k P$ or $I \models_1 \neg P$ and $I \models_{k-1} Q$.

Note that the *opt* function is not needed for basic choice formulas. To see why in the general case the more complicated definition is required consider the formula

$$F = (walking \times hotel{-}transport) \times public{-}transport$$

and take a model I satisfying $public{-}transport$ but neither $walking$ nor $hotel{-}transport$. According to 5' I satisfies the formula with degree 2, since I satisfies $public{-}transport$ with 1. But this seems unintuitive since $public{-}transport$ is only the 3rd-best option. According to 5 we obtain $I \models_3 F$.

Proposition 1 *Let F be a formula. There is a k such that $I \models_k F$ iff $I \models F^*$ where F^* is obtained from F by replacing each occurrence of $\times$ with standard disjunction ($\models$ is classical propositional consequence).*

Definition 4 *Let T be a set of formulas. An interpretation I is a model of T if it satisfies each formula in T to some degree.*

The degrees help us determine preferred models by ordering them lexicographically as follows:

Definition 5 *Let $M^k(T)$ denote the subset of formulas of T satisfied by a model M to degree k. A model M_1 is T-preferred over a model M_2 if there is a k such that $|M_1^k(T)| > |M_2^k(T)|$ and for all $j < k$: $|M_1^j(T)| = |M_2^j(T)|$. M is a preferred model of T iff M is a maximally T-preferred model.*

Intuitively, a preferred model of T is a model of T which satisfies the maximum number of best options of choice logic formulas. Note that *QCL* is a conservative extension of classical propositional logic: for a set of formulas T without appearance of $\times$ the preferred models of T coincide with the classical models of T.

We next define a consequence relation based on preferred models. We will restrict our attention here to classical conclusions, that is, the consequence relation we are interested in will be a relation between sets of formulas and classical formulas. The justification for this restriction lies in the intended use of *QCL*: we want to be able to derive properties describing a problem solution based on background knowledge, knowledge about the case at hand and choice formulas describing intended properties. Classical formulas are the formulas describing the intended problem solutions (e.g. the chosen hotel in the booking example).

Definition 6 *Let T be a set of formulas, and let A be a classical formula. $T \mid\!\sim A$ iff A is satisfied in all preferred models of T.*

This inference relation is obviously nonmonotonic. As an example consider $T = \{A \times B\}$. We have three models $\{A\}, \{A, B\}, \{B\}$ with satisfaction degree 1, 1, 2, respectively. This means that $\{A\}$ and $\{A, B\}$ are maximally preferred and we have $A \times B \mid\!\sim A$.

If we add $\neg A$ then the single model, and thus the single preferred model, is $\{B\}$. We thus obtain $\{A \times B, \neg A\} \mid\!\sim \neg A$. A is no longer a conclusion.

We now show that our inference relation satisfies properties usually considered intended in nonmonotonic reasoning.

Proposition 2 *The inference relation $\mid\!\sim$ satisfies cautious monotony, that is, $T \mid\!\sim A$ and $T \mid\!\sim B$ implies $T \cup \{A\} \mid\!\sim B$.*

Proposition 3 *The inference relation $\mid\!\sim$ satisfies cumulative transitivity, that is, $T \mid\!\sim A$ and $T \cup \{A\} \mid\!\sim B$ implies $T \mid\!\sim B$.*

In addition, we can show that $\mid\!\sim$ is rational, in the sense that it satisfies all rules of System P as well as rational monotony (not $T \mid\!\sim \neg A$ and $T \mid\!\sim B$ implies $T \cup \{A\} \mid\!\sim B$) [10].

3 Computation

In this section we investigate ways to compute consequences of *QCL* theories. The basic idea is to take a set of arbitrary formulas T and to proceed in 3 steps:

1. translate T to $Norm(T)$, a strongly equivalent normal form where all formulas containing $\times$ are basic choice formulas (to be defined below),

2. construct a stratified knowledge base $Skb(Norm(T))$ such that a classical formula F is lexicographically entailed by $Skb(Norm(T))$ iff $Norm(T) \mathrel{|\!\!\sim} F$,

3. use the techniques developed in [2] to generate $Extract(Skb(Norm(T)))$, a classical propositional knowledge base whose consequences are exactly the formulas lexicographically entailed by $Skb(Norm(T))$.

From the equivalence results described in the next subsections the following proposition is immediate:

Proposition 4 *$T \mathrel{|\!\!\sim} A$ iff $Extract(Skb(Norm(T))) \vdash A$.*

3.1 Translation to normal form

We first need a notion of equivalence:

Definition 7 *Let F_1, F_2 be formulas. F_1 is strongly equivalent to F_2, denoted $F_1 \doteq F_2$, iff $opt(F_1) = opt(F_2)$ and for all interpretations I and integers k we have $I \models_k F_1$ iff $I \models_k F_2$.*

The optionality of the formulas must be the same in order to guarantee that subformulas can be replaced. For instance, although $A \times A$ and A have the same satisfaction degree in all interpretations $A \times B$ and $A \times A \times B$ clearly have not. Note that for classical formulas without $\times$ all classical logical transformations can be performed, but for those with $\times$ some standard transformations are not valid: if A is classical, then $\neg\neg A$ is strongly equivalent to A, but $\neg\neg(A \times B)$ is not strongly equivalent to $A \times B$ due to the different optionalities of the formulas.

Proposition 5 *Ordered disjunction is associative, that is for arbitrary formulas F_1, F_2 and F_3 we have $((F_1 \times F_2) \times F_3) \doteq (F_1 \times (F_2 \times F_3))$.*

We want to translate QCL bases into a normal form consisting only of classical formulas and basic choice formulas which are defined as follows:

Definition 8 *A formula F is a basic choice formula if it is of the form $(A_1 \times \ldots \times A_n)$ where the A_i are classical propositional formulas and $n > 1$.*

For the translation to normal form we need the following strong equivalences:

Proposition 6 *Let $A_i, B_j, C, \ldots$ be formulas without $\times$. Then the following strong equivalences hold:*

1. $(A_1 \times \ldots \times A_n) \vee (B_1 \times \ldots \times B_m) \doteq (C_1 \times \ldots \times C_k)$
   where $k = max(m,n)$ *and* $C_i =$
   $(A_i \vee B_i)$ *if* $i \leq min(m,n)$
   A_i *if* $m < i \leq n$, *and*
   B_i *if* $n < i \leq m$.

   Example: $(A_1 \times A_2) \vee (B_1 \times B_2 \times B_3) \doteq (A_1 \vee B_1) \times (A_2 \vee B_2) \times B_3$

2. $(A_1 \times \ldots \times A_n) \wedge (B_1 \times \ldots \times B_m) \doteq (C_1 \times \ldots \times C_k)$
   where $k = max(m,n)$ *and* $C_i =$
   $[(A_1 \vee \ldots \vee A_i) \wedge B_i] \vee [A_i \wedge (B_1 \vee \ldots \vee B_i)]$ *if* $i \leq min(m,n)$
   $[(A_1 \vee \ldots \vee A_n) \wedge B_i]$ *if* $n < i \leq m$
   $[A_i \wedge (B_1 \vee \ldots \vee B_m)]$ *if* $m < i \leq n$.

   Example:
   $(A_1 \times A_2) \wedge (B_1 \times B_2 \times B_3) \doteq$
   $A_1 \wedge B_1 \times$
   $[(A_1 \vee A_2) \wedge B_2] \vee [A_2 \wedge (B_1 \vee B_2)] \times$
   $[(A_1 \vee A_2) \wedge B_3]$

3. $\neg(A_1 \times \ldots \times A_n) \doteq \neg(A_1 \vee \ldots \vee A_n)$

Repeated application of these transformation rules moves $\times$ outside (or eliminates it) until we obtain a classical formula or a basic choice formula:

Proposition 7 *Every formula F can be translated to a strongly equivalent formula F' which is either classical or a basic choice formula.*

We next show how QCL bases in normal form can be translated to stratified knowledge bases. For the results of the following section we need an additional condition: we say a set T of formulas is standardized iff different basic choice formulas do not possess syntactically *identical* prefixes[3] (they may, of course, be logically equivalent). This condition is added to avoid dealing with multi-sets in the following definitions. Of course, *any* theory T can be transformed into standardized form. This can simply be achieved by replacing identical formulas with some logically equivalent but syntactically different formula. For instance, a standardized form of $T = \{A \times B, A \times C\}$ can be $\{A \wedge A \times B, A \times C\}$.

A set of QCL formulas T is in normal form if it consists of classical or basic choice formulas only, and if it is standardized.

[3] $A_1 \times \ldots \times A_k$ is a prefix of $A_1 \times \ldots \times A_n$ whenever $k \leq n$.

3.2 Compilation to a stratified knowledge base

We recall the definition of a stratified knowledge base.

Definition 9 *A stratified knowledge base is a sequence $(K, S_1, \ldots, S_n)$ of sets of classical propositional formulas.*

Since we need K to be a non-default level we slightly modify the definition of lexicographically preferred subbases from [1, 13].

Definition 10 *Let $KB = (K, S_1, \ldots, S_n)$ be a stratified knowledge base. A maximal consistent subset S of $(K \cup S_1 \cup \ldots \cup S_n)$ is a lexicographically preferred subbase of KB iff*

1. *$K \subseteq S$, and*
2. *if S' is a maximal consistent subset of $(K \cup S_1 \cup \ldots \cup S_n)$ containing K and for some $k \in \{1, \ldots, n\} : |S' \cap S_k| > |S \cap S_k|$ then there is $j < k$ such that $|S \cap S_j| > |S' \cap S_j|$.*

The only difference between this definition and the original one [1, 13] is that there is no lexicographically preferred subbase if S_0 is inconsistent.

Definition 11 *Let $KB = (K, S_1, \ldots, S_n)$ be a stratified knowledge base. A formula F is lexicographically entailed by KB, denoted $KB \vdash_{lex} F$, iff F is entailed by all lexicographically preferred subbases of KB.*

We now define the translation:

Definition 12 *Let T be a QCL base in normal form. The stratified knowledge base associated with T, denoted $Skb(T)$, is*

$$Skb(T) = (T^*, T_1, \ldots, T_n)$$

where $n = max\{k \mid F \in T, opt(F) = k\} - 1$,
T^* is obtained from T by replacing each occurrence of $\times$ by $\vee$, and
$T_i = \{A_1 \vee \ldots \vee A_i \mid 1 \leq i < k, A_1 \times \ldots \times A_k \in T\}$.

Proposition 8 *$T \mid\!\sim F$ iff $Skb(T) \vdash_{lex} F$.*

Note that in the definition of T_i the case $i = k$ is not needed since the corresponding disjunctions are already in T^*. It turns out that there is a second, equivalent translation. In Def. 12 we can equivalently define

$$T_i = \{\neg A_1 \wedge \ldots \wedge \neg A_{i-1} \wedge A_i \mid 1 \leq i < k, A_1 \times \ldots \times A_k \in T\}.$$

To illustrate the translation consider again the hotel example given in the introduction (after $\neg hotel_1$ was learned). The stratified knowledge base associated with T is $Skb(T) = (T^*, T_1, T_2)$ where:

$$
\begin{aligned}
T^* = \{&\neg hotel_1,\\
&walking \vee hotel{-}transport \vee public{-}transport,\\
&hotel_1 \vee hotel_2 \vee hotel_3 \vee hotel_4,\\
&hotel_1 \rightarrow walking,\\
&hotel_2 \rightarrow \neg walking \wedge hotel{-}transport,\\
&hotel_3 \rightarrow \neg walking \wedge \neg hotel{-}transport \wedge public{-}transport,\\
&hotel_4 \rightarrow \neg walking \wedge \neg hotel{-}transport \wedge \neg public{-}transport\}\\
T_1 = \{&walking\}\\
T_2 = \{&walking \vee hotel{-}transport\}
\end{aligned}
$$

There is exactly one lexicographically preferred subbase from which we can prove $hotel_2$.

3.3 Compilation to a classical KB

This step is described in [2] and is not discussed here due to space limitations. We just mention that for the hotel example discussed above the compilation yields the classical knowledge base $T^* \cup \{walking \vee hotel{-}transport\}$.

4 Applications

Qualitative choice logic has numerous useful applications.

- In design and configuration intended properties can be described, ranked according to their desirability.
- In agent systems intended actions to be performed by agents can be specified together with backup actions covering situations where the standard actions are inappropriate.
- In database and web applications queries can be expressed which describe suboptimal results. For instance, one may start a query like: find a used convertible less than 2 years old in red, if red is unavailable then in black. This may be very useful for e-commerce applications.

Due to space limitations we cannot go into much detail in this abstract. This section will be extended in the final version of the paper.

5 Alternative definitions of entailment

In Sect. 2 we defined entailment for QCL in terms of a lexicographic ordering on models, based on the number of formulas satisfied to a certain degree.

This leads to an approach where solutions are preferred when they contain the highest number of most preferred options. For instance, if there are three choices with three options each, say

$$A_1 \times A_2 \times A_3, B_1 \times B_2 \times B_3, C_1 \times C_2 \times C_3$$

then a model M_1 satisfying A_1, B_1 and C_3 is preferred over a model M_2 satisfying A_2, B_2 and C_1 because the number of formulas satisfied in M_1 with degree 1 is 2, the number in M_2 is 1.

This may not be wanted for all applications. In our example we might consider M_2 a reasonable alternative: although it gives us only 1 best choice, we still get two second best options. The following strengthening of the preference relation is based on subsets rather than the number of formulas satisfied with a particular degree

Definition 13 *Let $M^k(T)$ denote the set of formulas of T satisfied by a model M of T to degree k. A model M_1 of T is T-inclusion-preferred over a model M_2 if there is a k such that $M_1^k(T)$ is a strict superset of $M_2^k(T)$ and for all $j < k$: $M_1^j(T) = M_2^j(T)$. M is an inclusion-preferred model of T iff M is a maximally T-inclusion-preferred model of T.*

Note that inclusion preference implies the cardinality based preference introduced earlier but not vice versa. We therefore get in general more maximally inclusion-preferred models and thus fewer conclusions. In the example above M_1 is not subset-preferred over M_2.

Another option is to just add up the (dis)satisfaction degrees of all formulas in T for each model and to prefer those models whose sum is minimal, that is whose overall dissatisfaction is minimal. In the example both M_1 and M_2 have overall degree 5 (1 + 1 + 3 and 2 + 2 + 1, respectively), thus none of the models is preferred to the other.

Even more fine grained distinctions could be introduced by adding to each option in an ordered disjunction some integer (increasing in value from left to right) which could be used to compute the overall dissatisfaction degree of models.

A further study of these alternative notions of entailment and corresponding compilation techniques is left for future work. A recent complexity result from Coste-Marquis and Marquis [8] may be relevant here: they prove that there is no way to compile any stratified knowledge base under inclusion preference in polynomial space whereas such a translation is possible under lexicographic preference by adding new propositional variables (the compiled base is "query equivalent" to the original one). This may be seen as an additional argument in favour of lexicographic preference.

6 Conclusions

We proposed in this paper a new nonmonotonic propositional logic for representing ranked options. The logic has a new connective called ordered disjunction. Since ordered disjunction is fully embedded in the language the ranking of the options may depend on the particular context, that is, it may depend on what else is true in the current situation.

There are numerous approaches to add preferences to nonmonotonic logics, for a discussion of some of them see [6]. Existing work on context dependent preferences in nonmonotonic logics is based on explicit representations of a preference ordering together with names for default rules [5, 7] or makes heavy use of meta-predicates [9]. The availability of ordered disjunction in QCL allows context dependent preferences among properties of a problem solution to be expressed much more conveniently.

We investigated computational aspects of QCL and showed how sets of formulas can be translated to stratified knowledge bases and, by results in [2], to classical propositional knowledge bases. We indicated a number of potential applications of the logic, and we presented alternative definitions of the consequence relation for applications where the lexicographic ordering based on the number of best possible options is not adequate.

In the full paper we will extend our discussion of potential applications and give more realistic examples. We will also compare our representation framework with recent proposals found in the literature. Poole [17] aims at a combination of logic and decision theory. His approach incorporates quantitative utilities whereas our preferences are qualitative. Interestingly, Poole uses a logic *without* disjunction ("rather than using disjunction ... we want to use probability and decision theory to handle uncertainty", Sect. 1.5) whereas we *enhance* disjunction. In [4] a graphical representation, somewhat reminiscent of Bayes nets, for conditional preferences among feature values under the *ceteris paribus* principle is proposed, together with corresponding algorithms. We are able to represent preferences among arbitrary formulas and developed a full fledged logic. In a series of papers [11, 19, 12], originally motivated by [3], the authors propose viewing conditional desires as constraints on utility functions. Intuitively, $D(a|b)$ stands for: the b-worlds with highest utility satisfy a. Our interpretation of ranked options is very different. Rather than being based on decision theory our approach can be viewed as giving a particular interpretation to the ceteris paribus principle.

In future work we want to investigate compilations of the alternative definitions of entailment, extensions of the inference relation to non-classical formulas, and the first order case.

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