A Short Introduction to Abstract Argumentation Frameworks

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November 2009
Argumentation in AI

Overview

- General idea: reasonable conclusions/decisions reached by
  1. constructing pro and con arguments
  2. evaluating arguments accordingly
- Different aspects: modeling the process, analyzing the conflicts, determining status, ...etc.
- Main distinction:
  1. Abstract argumentation frameworks: attack relations, semantics
  2. Deductive argumentation frameworks: logical structure of arguments
- Common interaction: deductive AFs instantiate abstract AFs and thus inherit semantics
Abstract Argumentation

- Arguments are “atomic”
- Argumentation frameworks (AFs) formalize relations (attacks) between arguments
- Semantics gives an abstract handle to solve the inherent conflicts between statements by selecting acceptable subsets

Deductive Argumentation

- Arguments are structured
- Often formulas together with supporting premises; conflicts based on contradictions
- Relationship to nonmonotonic logics
Abstract Argumentation

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Deductive Argumentation

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An argumentation framework (AF) is a pair \((A, R)\) where
- \(A\) is a set of arguments
- \(R \subseteq A \times A\) is a relation representing “attacks” (“defeats”)
Argumentation Frameworks

An argumentation framework (AF) is a pair $F = (A, R)$ where

1. $A$ is a set of arguments
2. $R \subseteq A \times A$ is a relation representing “attacks” (“defeats”)
Definitions (2)

Conflict-Free Set

Given an AF $F = (A, R)$. A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

Example

$\text{cf}(F) = \{\{a, c\}\}$,
**Conflict-Free Set**

Given an AF $F = (A, R)$. A set $S \subseteq A$ is **conflict-free** in $F$, if, for each $a, b \in S$, $(a, b) \not\in R$.

**Example**

\[ \text{cf}(F) = \{ \{a, c\}, \{a, d\} \}, \]
Definitions (2)

Conflict-Free Set

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A set $S \subseteq A$ is conflict-free in $F$, if, for each $a, b \in S$, $(a, b) \notin R$.

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$$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}\},$$
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Example

$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$
Admissible Set

Given an AF $F = (A, R)$. A set $S \subseteq A$ is **admissible** in $F$, if

- $S$ is conflict-free in $F$
- each $a \in S$ is defended by $S$ in $F$,
  - $a \in A$ is defended by $S$ in $F$, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

Example

![Diagram](image)

$adm(F) = \{ \{a, c\} \}$
Admissible Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in $F$, if

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Example

$adm(F) = \{\{a, c\}, \{a, d\}\}$,
Definitions (3)

Admissible Set

Given an AF \( F = (A, R) \). A set \( S \subseteq A \) is admissible in \( F \), if

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  - \( a \in A \) is defended by \( S \) in \( F \), if for each \( b \in A \) with \( (b, a) \in R \), there exists a \( c \in S \), such that \( (c, b) \in R \).

Example

\[
\text{adm}(F) = \{ \{a, c\}, \{a, d\}, \{b, d\} \},
\]

\[
\begin{align*}
\text{a} & \rightarrow \text{b} \\
\text{b} & \rightarrow \text{c} \\
\text{c} & \leftrightarrow \text{d} \\
\text{d} & \rightarrow \text{e}
\end{align*}
\]
Definitions (3)

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Example
$$adm(F) = \{ \{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset \}$$
**Grounded Extension**

The grounded extension of an AF $F = (A, R)$ is given by the least fixpoint of the operator $\Gamma_F : 2^A \rightarrow 2^A$, defined as

$$\Gamma_F(S) = \{ a \in A \mid a \text{ is defended by } S \text{ in } F \}$$

**Example**

$$\text{ground}(F) = \{ \{a\} \}$$
Definitions (5)

Preferred Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is preferred in $F$, if

- $S$ is admissible in $F$
- for each $T \subseteq A$ admissible in $T$, $S \not\subset T$

Example

\[ \text{pref}(F) = \{ \{a, c\}, \{a, d\}, \{a\}, \{c\}, \{d\}, \emptyset \} \]
Definitions (6)

Stable Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is stable in $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$.

Example

$$stable(F) = \{ \{a, c\} \}.$$
Stable Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is stable in $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$.

Example

$stable(F) = \{\{a, c\}, \{a, d\}\}$,
Definitions (6)

**Stable Extension**

Given an AF \( F = (A, R) \). A set \( S \subseteq A \) is **stable** in \( F \), if

- \( S \) is conflict-free in \( F \)
- for each \( a \in A \setminus S \), there exists a \( b \in S \), such that \( (b, a) \in R \).

**Example**

\[
stable(F) = \{ \{a, c\}, \{a, d\}, \{b, d\}, \}
\]
Stable Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is stable in $F$, if

- $S$ is conflict-free in $F$
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$.

Example

$$stable(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$$
Connections

- each AF has a unique grounded extension
- each (finite) AF has at least one preferred extension
- existence of stable extensions is not guaranteed
- grounded extension subset of intersection of preferred extensions
- each stable extension is preferred, but not vice versa