A Short Introduction to Abstract Argumentation Frameworks

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Argumentation in AI

Overview

- General idea: reasonable conclusions/decisions reached by
  1. constructing pro and con arguments
  2. evaluating arguments accordingly
- Different aspects: modeling the process, analyzing the conflicts, determining status, ...etc.
- Main distinction:
  1. Abstract argumentation frameworks: attack relations, semantics
  2. Deductive argumentation frameworks: logical structure of arguments
- Common interaction: deductive AFs instantiate abstract AFs and thus inherit semantics

Argumentation in AI (ctd.)

Abstract Argumentation

- Arguments are “atomic”
- Argumentation frameworks (AFs) formalize relations (attacks) between arguments
- Semantics gives an abstract handle to solve the inherent conflicts between statements by selecting acceptable subsets

Deductive Argumentation

- Arguments are structured
- Often formulas together with supporting premises; conflicts based on contradictions
- Relationship to nonmonotonic logics

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Argumentation Frameworks

An argumentation framework (AF) is a pair (A, R) where

- A is a set of arguments
- $R \subseteq A \times A$ is a relation representing "attacks" ("defeats")

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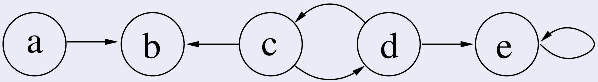
Example

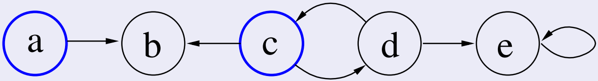


Definitions (2)

Conflict-Free Set

Given an AF $F = (A, R)$.
A set $S \subseteq A$ is conflict-free in F, if, for each $a, b \in S$, $(a, b) \notin R$.

Example



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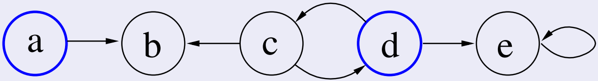

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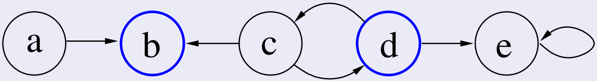

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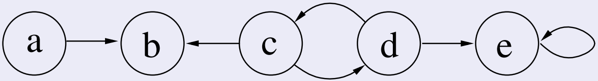

$cf(F) = \{\{a, c\}, \{a, d\}, \{b, d\}, \{a\}, \{b\}, \{c\}, \{d\}, \emptyset\}$

Definitions (3)

Admissible Set

Given an AF $F = (A, R)$. A set $S \subseteq A$ is admissible in F, if

- S is conflict-free in F
- each $a \in S$ is defended by S in F,
  - $a \in A$ is defended by S in F, if for each $b \in A$ with $(b, a) \in R$, there exists a $c \in S$, such that $(c, b) \in R$.

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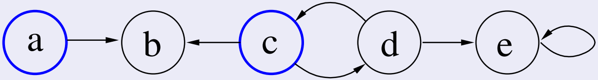

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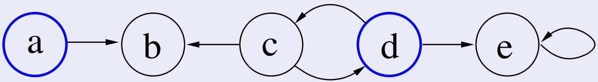

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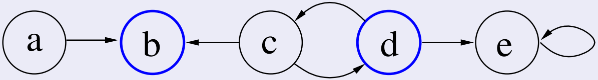

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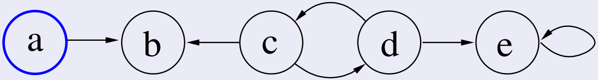


$adm(F) = \{\{a, c\}, \{a, d\}, \cancel{\{b, d\}}, \{a\}, \cancel{\{b\}}, \{c\}, \{d\}, \emptyset\}$

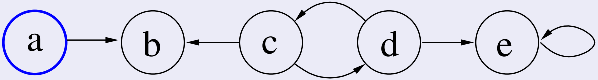
Definitions (4)

Grounded Extension

The grounded extension of an AF $F = (A, R)$ is given by the least fixpoint of the operator $\Gamma_F : 2^A \rightarrow 2^A$, defined as

$$\Gamma_F(S) = \{a \in A \mid a \text{ is defended by } S \text{ in } F\}$$

Example



$ground(F) = \{\{a\}\}$

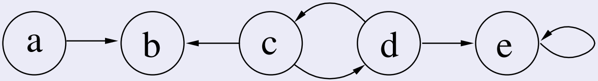
Definitions (5)

Preferred Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is preferred in F, if

- S is admissible in F
- for each $T \subseteq A$ admissible in T, $S \not\subset T$

Example



$pref(F) = \{\{a, c\}, \{a, d\}, \cancel{\{a\}, \{c\}, \{d\}, \emptyset}\}$

Definitions (6)

Stable Extension

Given an AF $F = (A, R)$. A set $S \subseteq A$ is stable in F, if

- S is conflict-free in F
- for each $a \in A \setminus S$, there exists a $b \in S$, such that $(b, a) \in R$.

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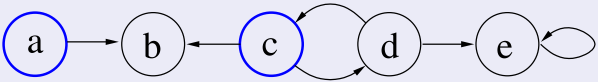

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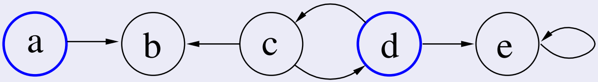

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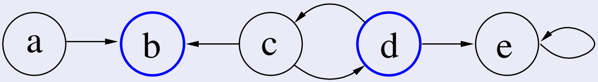

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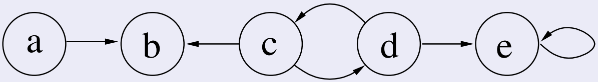

$\mathit{stable}(F) = \{\cancel{\{a, c\}}, \{a, d\}, \cancel{\{b, d\}}, \cancel{\{a\}}, \cancel{\{b\}}, \cancel{\{c\}}, \cancel{\{d\}}, \cancel{\emptyset}\}$

Connections

- each AF has a unique grounded extension
- each (finite) AF has at least one preferred extension
- existence of stable extensions is not guaranteed
- grounded extension subset of intersection of preferred extensions
- each stable extension is preferred, but not vice versa